19P1004

Reg. No .... Name ....

MSc DEGREE END SEMESTER EXAMINATION - OCTOBER 2019

SEMESTER 1 : PHYSICS

COURSE : **16P1PHYT01 : MATHEMATICAL METHODS IN PHYSICS - I**

(For Regular - 2019 Admission and Supplementary - 2016/2017/2018 Admissions)

Time : Three Hours Max. Marks: 75

Section A
Answer all Questions (1 mark each)

1. A vector **r** = x***i*** + y***j*** + z***k***. If **F** = r^n**r**, the value of $\nabla \times \mathbf{F}$ is
   (a) 0 (b) r (c) nr^{n-1} (d) 1
2. If AY = PY then Y =
   (a) PYA (b) PYA^{-1} (c) $A^{-1}PY$ (d) PYP^{-1}
3. The characteristic equation of matrix A is $\lambda^2 - \lambda - I = 0$, then
   (a) A^{-1} does not exist (b) A^{-1} exists (c) $A^{-1} = A + I$ (d) $A^{-1} = A - I$
4. Sort out the covariant component from among the following:
   (a) $\frac{\partial x_i}{\partial t}$ (b) $\frac{\partial u}{\partial x_j}$ (c) δ^i_j (d) none of these
5. The incorrect equation among the following is
   (a) $P_0(x) = 0$ (b) $P_1(x) = x$ (c) $P_n(-x) = (-1)^n P_n(x)$ (d) $P_n(-x) = (-1)^{n+1} P_n(x)$

(1 x 5 = 5)

Section B
Answer any 7 (2 marks each)

6. Express position and velocity of a particle in spherical polar coordinates.
7. What is a linear vector space?
8. Show that Pauli spin matrices anticommute in pairs.
9. Show that Eigen values of a Hermitian matrix are real and Eigen vectors are orthogonal.
10. State central limit theorem.
11. Find differential length dV in spherical polar coordinates.
12. Show that any tensor of rank 2 can be expressed as the sum of a symmetric and anti-symmetric tensor of rank 2.
13. What is Kronecker delta function? Give one application.
14. Show that $\Gamma(n+1) = n\Gamma n$ where n is an integer.
15. Prove that $P_n(1) = 1$

(2 x 7 = 14)

Section C
Answer any 4 (5 marks each)

16. Using Green's theorem evaluate $\int_c x^2 y dx + x^2 dy$ where c is the boundary described counter clockwise of the triangle with vertices (0, 0) , (1, 0) , (1, 1).

17. Find the inverse of the given matrix by Gauss–Jordan method:
$$\begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 2 \\ 1 & 1 & 3 \end{bmatrix}$$

18. Explain the differences between Binomial, Poisson and normal distributions.

19. What is the inner product of a tensor? Find the rank of the inner product of tensors $A^p{}_r$ and $B^{qs}{}_t$

20. Prove that Kronecker Delta is an invariant mixed tensor of rank 2.

21. Show that $x = 2 (J_1(x) + 3 J_3 (x) + 5 J_5(x) + \dots\dots\dots)$

(5 x 4 = 20)

Section D
Answer any 3 (12 marks each)

22.1. Define line, surface and volume integrals. Explain the theorems connecting these integrals

OR

2. State and prove Gauss' theorem and Stoke's theorem. Hence deduce Gauss law in electrostatics.

23.1. Determine the Eigen values and normalized Eigen vectors.
$$\begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

OR

2. What are Christoffel symbols? Drive transformation law for Christoffel symbol of first kind and show that they are not components of a tensor.

24.1. Write the Legendre's differential equation. Obtain the series solution of Legendre's differential equation.

OR

2. Show that y = Hn(x) is a solution of Hermite differential equation.

(12 x 3 = 36)