Reg. No .... Name **21P4016**

M. Sc. DEGREE END SEMESTER EXAMINATION - APRIL 2021

SEMESTER 4 : MATHEMATICS

COURSE : **16P4MATT17EL ; MULTIVARIATE CALCULUS AND INTEGRAL TRANSFORMS**

(For Regular - 2019 Admission and Supplementary - 2018/2017/2016 Admissions)

Time : Three Hours Max. Marks: 75

PART A
Answer All (1.5 marks each)

1. How Fourier integral differ from Fourier series.
2. Find the Laplace transform of $f(x) = e^{at}$
3. Find the Laplace transform of $sinhat$
4. Show that total derivative of a linear function is the function itself.
5. If $f(x) = ||x^2||$ then find $f'(c;u)$.
6. Let $f : \mathbb{R} \to \mathbb{R}^2$ be a function given by $f(t) = (\cos t, \sin t)$. Show that the ordinary Mean Value theorem does not hold in $[0, 2\pi]$.
7. Let $f : \mathbb{R} \to \mathbb{R}^2$ be defined by $f(t) = (\cos t, \sin t)$ and $f'(t)(u) = u(-\sin t, \cos t)$. Show that for every vector $a \in \exists`z' \in (0, 2\pi)$ such that $a.\{f(y) - f(x)\} = a.\{f'(z)(y-x)\}$.
8. Show by an example that f need not be one-one on S even when $J_f(x) \neq 0 \, \forall x \in S$.
9. Define the term support
10. Define the term primitive mapping

(1.5 x 10 = 15)

PART B
Answer any 4 (5 marks each)

11. Show that $\int_0^\infty \frac{x \sin ax}{1+x^2} dx = \frac{a}{|a|} \frac{\pi}{2} e^{-|a|}$, if $a \neq 0$.
12. Derive the exponential form of Fourier Integral Theorem
13. Assume f is differentiable at c with total derivative T_c. Then prove that the directional derivative $f'(c;u)$ exists for every u in R^n and we have $T_c(u) = f'(c;u)$.
14. Examine for extreme values $x^2 + y^2 + 6x + 12$
15. State and prove a global property of functions with nonzero Jacobian determinant.
16. Find $J_f(r, \theta, z)$ where $f(r, \theta, z)$ is defined by $x = rcos\theta, y = rsin\theta, z = z)$

(5 x 4 = 20)

PART C
Answer any 4 (10 marks each)

17.1. State and prove the convolution theorem for Fourier Transforms

OR

2. (i) Prove that $\frac{\Gamma(p)\Gamma(p)}{\Gamma(2p)} = 2\int_0^{\frac{1}{2}} x^{p-1}(1-x)^{p-1} dx$.

(ii) Make a suitable change of variable in (i) to obtain $\Gamma(2p)\Gamma\left({}^1/_2\right) = 2^{2p-1}\Gamma(p)\Gamma\left(p + \frac{1}{2}\right)$.

18.1. a)Derive the matrix form of Chain rule

b)Compute the gradient vector $\nabla f(x,y)$ at those points (x,y) in R^2 where it exists

$$f(x,y) = xysin\frac{1}{x^2+y^2} if f(x,y) \neq (0,0), f(0,0) = 0$$

OR

2. Assume that g is differentiable at a, with total derivative $g'(a)$. Let $b = g(a)$ and assume that f is differentiable at b, with total derivative $f'(b)$. Then prove that the composite function $h = fog$ is differentiable at a, and the total derivative $h'(a)$ is given by $h'(a) = f'(b)og'(a)$, the composition of the linear functions $f'(b)$ and $g'(a)$.

19.1. (a) State and prove second derivative test for extrema.

(b)Find and classify the extremum values of the function

f(x, y) = x^2 + y^2 + x + y + xy.

OR

2. State and prove a sufficient condition for differentiability

20.1. State and prove the theorem on partition of unity

OR

2. Prove that if ω and λ are k- and m- forms respectively of class C' in E, then $d(\omega \wedge \lambda) = (d\omega) \wedge \lambda + (-1)^k \omega \wedge d\lambda$ and if ω is of C'' in E, then $d^2\omega = 0$.

(10 x 4 = 40)