Reg. No .... Name **17P117**

M Sc DEGREE END SEMESTER EXAMINATION- NOVEMBER 2017

SEMESTER 1 : MATHEMATICS

COURSE : **16P1MATT02 ; BASIC TOPOLOGY**

(Common for Regular - 2017 / Supplementary - 2016 Admissions)

Time : Three Hours Max. Marks: 75

Section A
Answer all the questions (1.5 marks each)

1. Define discrete topology and trivial topology. When does a discrete topology coincide with a trivial topology?
2. When is a topological space said to be metrisable? Give an example.
3. Let (X, d) be a metric space and x,y distinct points in X. Show that there exists open sets U and V such that $x \in U, y \in V$ and $U \cap V = \phi$.
4. Prove that a constant function is always continuous.
5. Define a compact set A in a space $(X, \mathscr{T})$. Give an example of a set that is not compact.
6. Let $X = \{x_1, \ldots x_n\}$ be a topological space that contains only finitely many points.Is X compact? Explain.
7. Define a separated space with example.
8. Is the set of rational numbers connected ? Justify.
9. Define (i) T_2 space (ii) Completely regular Space.
10. Prove that compact subsets in a Hausdorff space are closed.

(1.5 x 10 = 15)

Section B
Answer any 4 (5 marks each)

11. Let X be a non empty set and $\mathscr{T} = \{G \subset X : X - G$is countable$\} \cup \{\phi\}$. Prove that $\mathscr{T}$ is a topology on X.
12. Define metric topology. Prove that every metric space is a topological space, where the topology is metric topology.
13. Prove that every second countable space is separable.
14. Define a qoutient map. Show that every closed, surjective map is a quotient map.
15. Differentiate connectedness and locally connectedness with an example.
16. Show that every Tychonoff space is regular.

(5 x 4 = 20)

Section C
Answer either 1 OR 2 of each question (10 marks each)

17.1. Let $\mathscr{C}$ be the family of all closed sets in a topological space $(X, \mathscr{T})$, then $\mathscr{C}$ has the following properties:
(i) $\phi \in \mathscr{C}, X \in \mathscr{C}$.
(ii)$\mathscr{C}$ is closed under arbitrary intersection.
(iii) $\mathscr{C}$ is closed under finite union.
Conversly, given any set X and a family $\mathscr{C}$ of its subsets which satisfies the above three properties, then there exist a unique topology $\mathscr{T}$ on X such that $\mathscr{C}$ coincides with the family of closed subsets of $(X, \mathscr{T})$.
OR

2. (a)Let X be a set, $\mathscr{T}$ a toplogy on X and S a family of subsets of X. Show that S is a subbase for $\mathscr{T}$ if and only if S generates $\mathscr{T}$.
(b) If $(X, \mathscr{T})$ is second countable and $Y \subset X$, then show that any cover of Y by members of $\mathscr{T}$ has a countable subcover.

18.1. (a) Let $[(X_i, \mathscr{T}_i), i = 1, 2, \ldots n]$ be a collection of topological spaces and $(X, \mathscr{T})$ their topological product. Prove that each projection π_i is continuous. Also show that if Z is any space then the fuction $f : Z \to X$ is continuous if and only if $\pi_i o f : Z \to X_i$ is continuous for all $i = 1, 2, \ldots n$.
(b) Sate and prove lebesgue covering lemma.
OR

2. (a) State and prove lebesgue covering lemma.(b)Prove that every second countable space is first countable.

19.1. (a) Prove that every closed and bounded interval is compact. (b) Show that union of collection of connected subsets of X having a common point is connected.
OR

2. (a) Prove that a subset of R is connected if it is an interval.(b) Prove that every closed and bounded interval is compact.

20.1. (a) Define T_4 and prove that all metric spaces are T_4.
(b)Prove that every regular lindeloff space is normal.
OR

2. (a)Show that the axioms T_0, T_1, T_2, T_3 and T_4 form a hierarchy of progressively stronger condition.
(b) Every continuous, one to one function from a compact space onto a Hausdorff space is an embedding.

(10 x 4 = 40)