Reg. No .... Name **17P103**

M Sc DEGREE END SEMESTER EXAMINATION- NOVEMBER 2017

SEMESTER 1 : MATHEMATICS

COURSE : **16P1MATT01 ; LINEAR ALGEBRA**

(For Regular - 2017 / Supplementary - 2016 Admissions)

Time : Three Hours Max. Marks: 75

Section A

Answer all the questions (1.5 marks each)

1. Is the set of vectors $\alpha = (a_1, \ldots, a_n) \in \mathbb{R}^n$ such that a_2 is rational a subspace of $\mathbb{R}^n$?
2. Show that the set of all 2×2 symmetric matrices over a field F is a subspace of the space of all $n \times n$ matrices over F.
3. Let V be the (real) vector space of all functions f from $\mathbb{R}$ into $\mathbb{R}$. Is the set of all functions f such that $f(-1) = 0$ a subspace of V?
4. Define linear functional. Give an example.
5. Describe the range and null space of the differentiation transformation defined on the vector space of polynomials of degree less than or equal to n.
6. Show that a linear tranformation from $T : \mathbb{R}^n \rightarrow \mathbb{R}^n$ is one-one if and only if it is onto.
7. Define alternating n-linear function.
8. Show that similar matrices have the same characteristic polynomial.
9. Define minimal polynomial for a linear operator T on a finite dimensional vector space V. State three properties which characterize the minimal polynomial.
10. Find a 3×3 matrix for which the minimal polynomial is x^2.

(1.5 x 10 = 15)

Section B

Answer any 4 (5 marks each)

11. Define subspace of a vector space V. Show that a non-empty subset W of V is a subspace of V if and only if for each pair of vectors α, β in W and each scalar c in F the vector $c\alpha + \beta$ is again in W.
12. Let V be a vector space which is spanned by a finite set of vectors $\beta_1, \ldots, \beta_m$.Show that any independent set of vectors in V is finite and contains no more than m elements.
13. Let W be the subspace of $\mathbb{R}^5$ which is spanned by the vectors $(2, -2, 3, 4, -1), (0, 0, -1, -2, 3), (-1, 1, 2, 5, 2), (1, -1, 2, 3, 0)$. Determine a basis for the annihilator of W°.
14. Show that $\{(1, 2), (3, 4)\}$ is a basis for $\mathbb{R}^2$. Let T be the unique linear transformation from $\mathbb{R}^2$ to $\mathbb{R}^3$ such that $T(1, 2) = (3, 2, 1)$ and $T(3, 4) = (6, 5, 4)$. Find $T(1, 0)$
15. Let A be an $n \times n$ matrix with λ as an eigen value. Show that,
(a) $k + \lambda$ is an eigen value of $A + kI$.
(b) If T is non-singular, $\frac{1}{\lambda}$ is an eigen value of A^{-1}.

16. Let V be a finite-dimensional nvector space over the field F and let T be a linear operator on V. Prove that T is diagonalizable if and only if the minimal polynomial for T is of the form $(x-c_1)(x-c_2)\dots(x-c_k)$, where the $c_i \in F$ are distinct.

(5 x 4 = 20)

Section C

Answer either 1 OR 2 of each question (10 marks each)

17.1. Let W be the subspace of $\mathbb{C}^3$ spanned by $\alpha_1 = (1,0,i)$ and $\alpha_2 = (1+i,1,-1)$.
(a) Show that α_1 and α_2 form a basis for W.
(b) Show that the vectors $\beta_1 = (1,1,0)$ and $\beta_2 = (1,i,1+i)$ are in W and form another basis for W.
(c) What are the coordinates of α_1 and α_2 in the ordered basis $\{\beta_1, \beta_2\}$ for W?
OR

2. Let V be the vector space of all 2×2 matrices over the field F.Let W_1 be the set of matrices of the form $\begin{bmatrix} x & -x \\ y & z \end{bmatrix}$ and let W_2 be the set of matrices of the form $\begin{bmatrix} a & b \\ -a & c \end{bmatrix}$.Prove that W_1 and W_2 are subspaces of V. Also find the dimensions of W_1, W_2,$W_1 + W_2$ and $W_1 \cap W_2$.

18.1. Let V be a finite dimensional vector space over the field F, and let W be a subspace of V.
(a) Show that $dim\, W + dim\, W^\circ = dim\, V$.
(b) Show that if W is a k-dimensional subspace of an n-dimensional vector space V, then W is the intersection of $(n-k)$ hyperspaces in V.
OR

2. (a) Let V be an n-dimensional vector space over the field F and let W be an m-dimensional vector space over F. Prove that $L(V,W)$ is finite dimensional and has dimension mn.
(b) Let f is a non-zero linear functional on the vector space V. Prove that the null space of f is a hyperspace in V. Also prove that every hyperspace in V is the null spce of a non-zero linear functional on V.

19.1. (a) Let T and U be linear operators on the finite dimensional vector space V. Prove that
(i) det $(TU) =$ (det T) (det U)
(ii) Define orthogonal matrix.If A is orthogonal, show that det $A = \pm 1$. Give an example of an orthogonal matrix for which det $A = -1$.
(b) If A is an invertible $n\times n$ matrix over a field F, show that det $A \neq 0$.
OR

2. Let A be an $n \times n$ matrix over the field F. Show that A is invertible over F if and only if det $A \neq 0$.When A is invertible, show that
$A^{-1} = [det(A)]^{-1}$. Adj A, where Adj A is the adjoint of A.

20.1. Let T be a linear operator on a finite-dimensional vector space V. Let $c_1, c_2, \dots, c_k$ be the distinct characteristic values of T and let W_i be the null space of $(T - c_i I)$. Show that the following are equivalent.
(a) T is diagonalizable.
(b) The characteristic polynomial for T is $f = (x-c_1)^{d_1}\dots(x-c_k)^{d_k}$ and dim $W_i = d_i, i = 1,2,\dots,k$.
(c) dim W_1+. . .+ dim W_k=dim V

OR

2. Let T be a linear operator on the finite dimensional vector space V. Let $c_1, \ldots, c_k$ be the distinct characteristic values of T and let W_i be the characteristic space associated with the characteristic value c_i. If $W = W_1 + \ldots + W_2 + \ldots + W_k$, show that dim W = dimW_1+ dim$W_2 + \ldots +$ dimW_k.

(10 x 4 = 40)