Reg. No .... Name **20P1057**

M. Sc DEGREE END SEMESTER EXAMINATION - OCT 2020 : FEBRUARY 2021

SEMESTER 1 : MATHEMATICS

COURSE : **16P1MATT05 : COMPLEX ANALYSIS**

(For Regular - 2020 Admission and Supplementary - 2016/2017/2018/2019 Admissions)

Time : Three Hours Max. Marks: 75

PART A

Answer any 10 (1.5 marks each)

1. Prove that an analytic function with constant modulus reduces to a constant.
2. Show that $w = iz + i$ maps half plane $x > 0$ onto the halfplane $v > 1$
3. Prove that the map $w = \bar{z}$ is not conformal.
4. State the fundamental theorem of algebra
5. Evaluate $\int_r xdz$, where r is the directed line seqment from 0 to $1 + i$
6. $r(\bar{t}) = t^2 e^{i\pi/4}, t \in (0, 1]$ is a non simple smooth contour. True or false. Justify
7. State Taylor's theorem
8. Explain locally exact differentials with examples.
9. State Schwarz reflection principle.
10. Find the residue at $z = 0$ of the function $(f(z) = \frac{e^{2z}}{z^4}$

(1.5 x 10 = 15)

PART B

Answer any 4 (5 marks each)

11. Define cross ratio and prove that it remains invarient under a linear transformation
12. Find a bilinear transformation which maps the points $0, -i, -1$ onto the points $i, 1, 0$
13. State and prove Liouville's theorem
14. Let z_j be the zeros of a function $f(z)$ which is analytic in a disk Δ and does not vanish identically, each zero being counted as many times as its order indicates. Then prove that for every closed curve γ in Δ which does not pass through a zero $\sum_j n(\gamma, z_j) = \frac{1}{2\pi i}$ $\int_\gamma \frac{f^{'}(z)}{f(z)} dz$, where the sum has only a finite number of terms $\neq 0$
15. State Rouche's theorem and apply it to determine the number of roots of the equation $z^8 - 4z^5 + z^2 - 1 = 0$
16. Find the residues of $f(z) = \frac{1}{z(z^2+3)(z^2+2)^3}$ at its poles

(5 x 4 = 20)

PART C

Answer any 4 (10 marks each)

17.1. Discuss the transformation $w = \frac{1}{z}$?. Also find the images of the infinite strips

$(i) 1/4 < y < 1/2$ $(ii) 0 < y < 1/2$

$(iii) 1/4 < x < 1/2$

OR

2. Find the Mobius transformation which maps the circle $|z| \leqq 1$ on $|w-1| \leqq 1$ and makes the points $z = 0, 1$ correspond to $w = 1/2, 0$ respectively

18.1. State and prove Cauchy's theorem for a rectangle.

OR

2. a. Evaluate $\int_{|z|=5} \frac{z+5}{z^2-3z-4} dz$

b. State and prove Morera's theorem.

19.1. State and give the topological and analytic proof of Maximum principle

OR

2. Suppose that $f(z)$ is analytic at z_0, $f(z_0) = w_0$ and that $f(z) - w_0$ has a zero of order n at z_0. Then prove that if $\xi > 0$ is sufficiently small, $\exists$ a corresponding $\delta > 0$ such that for all a with $|a - w_0| < \delta$ the equation $f(z) = a$ has exactly n roots in the disk $|z - z_0| < \xi$.

20.1. State and prove Poisson's Integral formula

OR

2. Evaluate $\int_0^{\pi} \frac{d\theta}{a^2 + sin^2\theta}$ where $a > 0$

(10 x 4 = 40)