Reg. No .... Name **21P2031**

M. Sc DEGREE END SEMESTER EXAMINATION - JULY 2021

SEMESTER 2 : MATHEMATICS

COURSE : **16P2MATT08 : ADVANCED COMPLEX ANALYSIS**

(For Regular - 2020 Admission and Supplementary - 2019/2018/2017/2016 Admissions)

Time : Three Hours Max. Marks: 75

PART A
Answer All (1.5 marks each)

1. Write Hadamard's Formula for the radius of convergence
2. Prove that $\left|\frac{1}{2} = \right|\pi$
3. State Abel's limit theorem.
4. State Hadamard's theorem
5. State Arzela's Theorem
6. Define Reimann's zeta function.
7. Define a free boundary arc.
8. State Schwarz-- Christoffel formula.
9. Define a doubly periodic function.
10. Define a period module.

(1.5 x 10 = 15)

PART B
Answer any 4 (5 marks each)

11. Represent $\sin \pi z$ in the form of canonical product.
12. If $\lim_{n\to\infty} z_n = A$. Prove that $\lim_{n\to\infty} \frac{1}{n}(z_1 + z_2 + \ldots + z_n) = A$.
13. State and prove Jenson's formula.
14. Show that a family $\mathcal{F}$ is normal if and if its closure $\bar{\mathcal{F}}$ w.r.t the distance function $\rho(f,g) = \sum_{k=1}^{\infty} \frac{\delta_k(f,g)}{2^k}$ is compact.
15. State and prove Harnack's inequality.
16. Show that $\mathcal{P}(z) - \mathcal{P}(u) = \frac{-\sigma(z-u)\sigma(z+u)}{(\sigma(z))^2(\sigma(u))^2}$.

(5 x 4 = 20)

PART C
Answer any 4 (10 marks each)

17.1. Define the Gamma function. Prove that $|z = \frac{e^{-\gamma z}}{z} \prod_{1}^{\infty}(1 + \frac{z}{n})^{-1} e^{zh}$. Also show that $|z\left|z + \frac{1}{2} = e^{az+b}\right|2z$

OR

2. State and prove the Weierstears Theorem for canonical product

18.1. State and prove `Laurent Theorem'

OR

2. Derive Poisson-Jensen formula

19.1. State and prove Reimann Mapping theorem.

OR

2. Show that any even elliptic function with periods w_1, w_2 can be expressed in the form $C\prod_{k=1}^{n}\frac{\mathcal{P}(z)-\mathcal{P}(a_k)}{\mathcal{P}(z)-\mathcal{P}(b_k)}$, provided 0 is neither a zero nor a pole. What is the corresponding form if the function either vanishes or becomes infinite at origin.

20.1. a. Prove that $\frac{\sigma'(z)}{\sigma(z)} = \zeta(z)$

b. Prove that $\sigma(-z) = -\sigma(z)$

OR

2. a) Prove that $\begin{vmatrix} \mathcal{P}(z) & \mathcal{P}'(z) & 1 \\ \mathcal{P}(u) & \mathcal{P}'(u) & 1 \\ \mathcal{P}(u+z) & -\mathcal{P}'(u+z) & 1 \end{vmatrix} = 0$

b) Prove that $\mathcal{P}(2z) = \frac{1}{4}\left[\frac{\mathcal{P}"(z)}{\mathcal{P}'(z)}\right]^2 - 2\mathcal{P}(z)$.

(10 x 4 = 40)