Q.Code…………………… Name:…………………………………………
Reg. No……………………………………..

B. Sc DEGREE END SEMESTER EXAMINATION OCTOBER 2017

SEMESTER – **5: MATHEMATICS**

COURSE: **15U5CRMAT7, ABSTRACT ALGEBRA**

Time: Three Hours Maximum: 75 Marks

Part A

Each Question carries 1 Mark

Answer All Questions

1. Find the identity element in the binary structure $< Q,*>$ if it exists when $a * b = ab/7$ for all $a, b \in \mathbb{Q}$.
2. Express the additive inverse of 21 in the group $< \mathbb{Z}_{73}, +_{73} >$ as a positive integer in $\mathbb{Z}_{73}$.
3. Fill in the blanks: Order of the group of symmetries of a rectangle is ________.
4. Define a cyclic group. Give an example of a non-cyclic group.
5. Define index of a subgroup H in a group G.
6. Every homomorphism is an isomorphism. State True/ False. Justify your answer.
7. How many elements are there in the ring of matrices $M_2(\mathbb{Z}_3)$?
8. Define a division ring.
9. The Characteristic of $n\mathbb{Z}$ is n. State True/ False. Justify your answer
10. Is $-i$ a unit in the ring of Gaussian integers $\{a + ib : a, b \in \mathbb{Z}\}$? Justify your answer.

Part B

Each Question carries 2 Marks

Answer any Eight

11. Show that arbitrary intersection of subgroups is a subgroup.
12. The Dihedral group D_3 has a cyclic subgroup of order 3 contained in it. State True/ False. Justify your answer.
13. Define the order of an element a in a group G. Give example for an element in S_7 with the largest possible order?
14. Is -1 the generator of the cyclic group $\mathbb{Z}$? If yes, describe how to generate 3 using the generator -1.
15. Does there exist an element of order in 4 in in $\mathbb{Z}_{14}$? Justify your answer.
16. Prove that every cyclic group is abelian.
17. Find all the units in the ring $\mathbb{Z}_{12}$.

18. Show that every field is an integral domain.
19. Define Ideal of a ring R with suitable examples.
20. Define characteristic of an integral domain with a suitable example.

Part C
Each Question carries 5 Marks
Answer Any Five

21. Show that the subset S of $M_n(\mathbb{R})$ consisting of all invertible $n \times n$ matrices under matrix multiplication is a group.
22. Draw the group table of a cyclic group of order 5.
23. Define normal subgroup with suitable examples. If H is a subgroup of index 2 in a finite group G, then show that $H \trianglelefteq G$.
24. Define an automorphism of a group. Show that all automorphisms of a group G form a group under function composition.
25. Show that the Characteristic of an integral domain is either 0 or $a\ prime\ number$.
26. Prove that a field contains no proper non-trivial ideals.
27. Differentiate between left ideal and right ideal of a ring R. Show that for a field F, the set of all matrices of the form $\begin{pmatrix} a & b \\ 0 & 0 \end{pmatrix}$ for $a, b \in F$ is a right ideal but not a left ideal of $M_2(F)$.

Part D
Each Question carries 12 Marks
Answer Any Two

28. i) Show that there exists a homomorphism ϕ: $S_n \to \mathbb{Z}_2$ which maps every odd permutations to 1. Also find the $Kernel$ of the homomorphism.
    ii) State and prove the necessary and sufficient condition for a non-empty subset H of a group G to be a subgroup of G.
29. State and prove Cayley's theorem.
30. State and prove fundamental theorem for group homomorphism.
31. Show that if $< N, +>$ is an additive subgroup of the additive group $< R, +>$ of a ring R, then the operations of induced addition and multiplication are both well defined on the cosets $r + N$ for $r \in R$ if and only if $rN \subseteq N$ and $Nr \subseteq N$ for all $r \in R$.

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