Reg. No.. Name.. **15U514**

B.Sc. DEGREE END SEMESTER EXAMINATION OCTOBER 2017

SEMESTER **–5: MATHEMATICS (CORE COURSE)**

COURSE: **15U5CRMAT6-15U5CRCMT6: DIFFERENTIAL EQUATIONS**

[Common for B.Sc. Mathematics and B.Sc. Computer Applications]

(For Regular 2015 admission)

Time: Three Hours Max. Marks: 75

PART A

Answer **all** questions. Each question carries 1 mark.

1. Find the value of b for which the equation $(xy^2 + bx^2y)dx + (x + y)x^2dy = 0$ is exact.
2. Solve the differential equation $y' = 1 + y^2$.
3. Write the Bernoulli's differential equation.
4. Find the Wronskian of $e^{\lambda_1 x}$, $e^{\lambda_2 x}$.
5. Verify that the function $y = \sqrt{2}\, cosx + 9\sin x$ is a solution of the homogeneous linear differential equation $y'' + y = 0$.
6. Solve $y'' - 4y' + 4y = 0$.
7. Write Bessel's equation.
8. Find the singular points of the differential equation $2x^2\frac{d^2y}{dx^2} - x\frac{dy}{dx} + (x - 5)y = 0$.
9. Form a partial differential equation by eliminating the arbitrary constants from $z = ax + by + a^2 + b^2$.
10. Write the general solution of Lagrange's equation. (1 x 10 = 10)

PART B

Answer **any eight** questions. Each question carries 2 marks.

11. Prove that $\mu(x, y) = x$ is an integrating factor of the differential equation $(3xy + y^2) + (x^2 + xy)y' = 0$.
12. Obtain the general solution of the equation $16y'' - 8y' + 145y = 0$.
13. Find the orthogonal trajectories of the family of parabolas $y = cx^2$.
14. Given that e^{-x}, e^{3x} and e^{4x} are all solutions of $y''' - 6y'' + 5y' + 12y = 0$. Show that they are linearly independent on the interval $-\infty < x < \infty$ and write the general solution.
15. Solve the equation $x\, siny\, dx + (x^2 + 1)cosy\, dy = 0$.
16. Find the general solution of $\frac{d^4y}{dx^4} + \frac{d^2y}{dx^2} = 0$.
17. Show that $J_0(kx)$, where k is a constant satisfies the differential equation $x\frac{d^2y}{dx^2} + \frac{dy}{dx} + k^2xy = 0$.

18. Transform the single linear differential equation $\frac{d^2x}{dt^2} - 3\frac{dx}{dt} + 2x = t^2$ into a system of first order differential equations.
19. Write a set of parametric equations of a surface $x^2 + y^2 + z^2 = a^2$.
20. Find a differential equation of all spheres of fixed radius having their centres in the xy plane. (2 x 8 = 16)

PART C

Answer **any five** questions. Each question carries 5 marks.

21. Solve the initial value problem $(x^2+1)\frac{dy}{dx} + 4xy = x, y(2) = 1$.
22. Solve $(x + 2y + 3)dx + (2x + 4y - 1)dy = 0$.
23. Given that $y = x$ is a solution of $(x^2+1)\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 2y = 0$ then find a linearly independent solution by reducing the order.
24. Solve $\frac{d^3y}{dx^3} + \frac{dy}{dx} = 2x^2 + 4\, sinx$.
25. Find a power series solution in powers of x of the differential equation $y'' + xy' + y = 0$.
26. Show that $\frac{d}{dx}[x^n J_n(x)] = x^n J_{n-1}(x)$.
27. Find the general integral of $x(y-z)p + y(z-x)q = z(x-y)$. (5 x 5 = 25)

PART D

Answer **any two** questions. Each question carries 12 marks.

28. Define an oblique trajectory. Find a family of oblique trajectories that intersect the family of straight lines $y = cx$ at angle 45^0.
29. Find the particular integral of the equation $(x^2+1)\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 2y = 6(x^2+1)^2$ by the method of variation of parameters , given that $y = x$ and $y = x^2 - 1$ are linearly independent solutions of the corresponding homogeneous equation.
30. Solve the Bessel's equation of order p.
31. (i) Find the integral curves of the equation $\frac{dx}{x+z} = \frac{dy}{y} = \frac{dx}{z+y^2}$.

    (ii) Find the general solution of the differential equation $x^2\frac{\partial z}{\partial x} + y^2\frac{\partial z}{\partial y} = (x+y)z$.

(12 x 2 = 24)

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