Reg. No .... Name **21P2018**

M. Sc DEGREE END SEMESTER EXAMINATION - JULY 2021

SEMESTER 2 : MATHEMATICS

COURSE : **16P2MATT07 : ADVANCED TOPOLOGY**

(For Regular - 2020 Admission and Supplementary 2019/2018/2017/2016 Admissions)

Time : Three Hours Max. Marks: 75

PART A

Answer any 10 (1.5 marks each)

1. State the Urysohn characterization of normality.
2. Give an example of a sequence of functions which is point wise convergent but not uniformly convergent.
3. Define a box and a wall in πX_i.
4. Prove that the evaluation function of a family of functions is one-one iff that family distinguishes points.
5. Give an example of a metric space which is not second countable.
6. If every net in a space X can converge to any point in it, then prove that X is indiscrete.
7. Define an ultra filter on a set X.
8. Let A be a subset of X and let $x \in X$. If $x \in \bar{A}$. Prove that there exists a net in A which converges to x.
9. Define a sequentially compact space. Give an example
10. Let $X^+ = XU\{\infty\}$ be the one point compactification of the space X. Prove that if $\{\infty\}$ is open in X^+, then X is compact.

(1.5 x 10 = 15)

PART B

Answer any 4 (5 marks each)

11. State Urysohn's Lemma and using it, prove that every T_4-space is Tychonoff.
12. Define projection functions and prove that they are open but not closed.
13. Prove that a topological space X is completely regular iff the family of all continuous functions from X into $[0, 1]$ distinguishes points from closed sets.
14. Prove that a space is compact iff every family of closed subsets of it, which has the f.i.p., has a non-empty intersection.
15. Prove that $\mathcal{F}$ is an ultra filter on a set X iff for any $A \subset X$ either $A \in \mathcal{F}$ or $X - A \in \mathcal{F}$.
16. Prove that a first countable, countably compact space is sequentially compact.

(5 x 4 = 20)

PART C

Answer any 4 (10 marks each)

17.1. If the product space is non-empty, prove that each co-ordinate space is embeddable in it and hence prove that if a topological product is T_0, T_1, T_2 or regular, then each co-ordinate space has the corresponding property.

OR

2. Prove that a product space is locally connected iff each co-ordinate space is locally connected and all except finitelymany of them are connected.

18.1. State and prove Tychonoff Embedding theorem.

OR

2. Define the evaluation function \`e' of the indexed family of functions. $\{f_i : X \to Y_i; i \in I\}$ and prove that it is the only function from X into πY_i such that $\pi_i \circ e = f_i; \forall i \in I$. Also prove that e is 1-1 iff the family $\{f_i\}$ distinguishes points and e is continuous iff each f_i is continuous.

19.1. For a topological space X, prove that the following statements are equivalent.

a. X is compact
b. Every net in X has a cluster point in X.
c. Every net in X has a convergent subnet in X.

OR

2. For a filter $\mathcal{F}$ on a set X, prove that the following statements are equivalent.

a. $\mathcal{F}$ is an ultra filter.
b. For any $A \subset X$ either $A \in \mathcal{F}$ or $X - A \in \mathcal{F}$.
c. For any $A, B \subset X, A \cup B \in \mathcal{F}$ iff either $A \in \mathcal{F}$ or $B \in \mathcal{F}$

20.1. Prove that countable compactness is preserved under continuous function and a countably compact metric space is compact. Also prove that every continuous, real-valued functions on a countably compact space is bounded and attains its extrema.

OR

2. Prove that a subspace of a locally compact, Hausdorff space is locally compact iff it is open in its closure.

(10 x 4 = 40)