Reg. No.. Name.. **15U504**

B.Sc. DEGREE END SEMESTER EXAMINATION OCTOBER 2017

SEMESTER **–5: MATHEMATICS (CORE COURSE)**

COURSE: **15U5CRMAT5-15U5CRCMT5: MATHEMATICAL ANALYSIS**

[Common for B.Sc. Mathematics and B.Sc. Computer Applications]

(For Regular 2015 admission)

Time: Three Hours Max. Marks: 75

PART A

Answer **all** *questions. Each question carries* 1 *mark.*

1. State Archimedian property of real numbers.
2. Find the supremum of $\{2+\frac{1}{n} : n \in N\}$.
3. Give an example of an open set which is not an interval
4. Find the closure of a set **N** of natural numbers.
5. Give an example of a set which is neither open nor closed
6. Give an example of a sequence which oscillates infinitely.
7. Find the derived set of the set of all rational numbers.
8. Give an example of a sequence which is not monotonic.
9. Check whether z = 1+i satisfies the equation $z^2 - 2z + 2 = 0$.
10. Reduce $(1 - i)^4$ into a real number. (1 x 10 = 10)

PART B

Answer ***any*** **eight** *questions. Each question carries* 2 *marks.*

11. Let S and T be nonempty bounded subsets of **R** with $S \subset T$. Prove that

    $\inf T \le \inf S \le \sup S \le \sup T$
12. Show that the set S = {1, -1, 1/2, -1/2 , 1/3, -1/3, …} is neither open nor closed .
13. Show that set of all integers is countable
14. Prove that derived set of every finite set is empty.
15. Show that union of two closed sets is closed.
16. Show that if $\{x_n\}$ converges to L, then $\{|x_n|\}$ converges to $|L|$.
17. Prove that every monotonic increasing sequence which is not bounded above diverges to infinity
18. Show that the sequence $\{(a_n)^{1/n}\}$ converges where $a_n = \frac{n^n}{(n+1)(n+2).......(n+n)}$.
19. Prove that $|z_1+z_2|^2 \le |z_1|^2+|z_2|^2$ for any two complex numbers z_1 and z_2.
20. Sketch the set $|2z+5| > 3$ where z is a complex number. (2 x 8 = 16)

PART C

Answer ***any*** **five** *questions. Each question carries* 5 *marks.*

21. Prove that infimum of a bounded set is always a member of its closure
22. Prove that the set of real numbers in (0,1) is uncountable .

23. Prove that closure of a bounded set is bounded.
24. Show that every convergent sequence is bounded. Is the converse true? Justify your answer.
25. Show that $\{S_n\}$ where $S_n = (1+ 1/n)^n$ is convergent and show that its limit lies between 2 and 3
26. Prove that a Cauchy sequence in **R** is always convergent.
27. Use De-Moivre's formula to derive $\sin 3\theta = 3\cos^2\theta \sin\theta - \sin^3\theta$. (5 x 5 = 25)

PART D

Answer ***any*** **two** *questions. Each question carries* 12 *marks.*

28. Prove the equivalence of Dedekind's property and order completeness property of real numbers.
29. a) Prove that a set is closed iff its complement is open.
    b) If S and T are subsets of real numbers, then show that $(S \cup T)' = S' \cup T'$.
30. State and prove Bolzano-Weierstrass theorem for sequences. Is its converse true? Justify.
31. a) Let $\{a_n\}$ be a sequence such that $\lim \frac{a_{n+1}}{a_n} = L$. Then show that $\lim a_n = 0$ if $|L| < 1$ and $\lim a_n = \infty$ if $L > 1$.

    b) Show that for any real x, $\lim \frac{x^n}{n!} = 0$. (12 x 2 = 24)

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