Reg. No .... Name **21P2004**

M. Sc DEGREE END SEMESTER EXAMINATION - APRIL 2021

SEMESTER 2 : PHYSICS

COURSE : **16P2PHYT05 : MATHEMATICAL METHODS IN PHYSICS- II**

(For Regular - 2020 Admission and Supplementary 2019/2018/2017/2016 Admissions)

Time : Three Hours Max. Marks: 75

PART A
Answer All (1 mark each)

1. The value of the integral $I = \frac{1}{2\pi i}\oint_c \frac{dz}{z-3}$ where c is the circle $|z| = 1$ is
   (a) 1 (b) 1/2 (c) 2 (d) 0
2. If a set of vectors are said to be linearly independent then ....
   (a) one of the vectors cannot be expressed with any of the rest
   (b) their Gram determinant is zero
   (c) both of these
   (d) no conclusion may be drawn out of the given statement.
3. $F(\omega + ia)$ is the Fourier transform of ....
   (a) $e^{at}f(t)$ (b) $e^{-at}f(t)$ (c) $e^{at}f(t/a)$ (d) $e^{-at}f(t/a)$
4. The Fourier transform of a Gaussian is ....
   (a) Poissonian (b) Lorentzian (c) Laplacian (d) Gaussian
5. The solution of one dimensional heat equation $\frac{\partial u}{\partial t} = \alpha^2 \frac{\partial^2 u}{\partial x^2}$ exist, if
   (a) RHS is a constant (b) both LHS and RHS are constant (c) LHS is a constant (d) all of these

(1 x 5 = 5)

PART B
Answer any 7 (2 marks each)

6. State and explain Cauchy's residue theorem.
7. Express the function $f(z) = (z^*)^2/z$ in the form $u(x, y) + iv(x, y)$.
8. Prove that all elements in a class of any group have the same trace.
9. What are discrete and continuous groups?
10. What is the Laplace transform of sinh(t)?
11. Show that Fourier transform is a linear operation.
12. What is meant by convolution theorem of Laplace transforms.
13. Write Laplace's and Poisson's equation. Express them in cartesian coordinates.
14. Explain the any two PDEs relevant in Physics.
15. Find the solution of one dimensional Laplace equation in Cartesian coordinates.

(2 x 7 = 14)

PART C
Answer any 4 (5 marks each)

16. Given w(x,y) = u(x,y)+iv(x,y). If u and v are real functions and if w is analytic, show that $\nabla^2 u = \nabla^2 v = 0$.
17. Show that the three cube roots of unity form an abelian group under multiplication.
18. Develop the Fourier sine transform of a finite wave train.
19. Find the fourier cosine transform of f(t) = e^{-at}
20. Show that Green's function is symmetric with respect to its two variables.
21. Express Poisson's equation in spherical polar coordinates.

(5 x 4 = 20)

PART D

Answer any 3 (12 marks each)

22.1. Find the Laurent series of $f(z) = \frac{1}{z(z-2)^3}$ about the singularities z=0 and z=2, separately. Find the residues at each pole.

OR

2. Show that $\int_0^\infty \frac{dx}{(1+x^4)} = \frac{\pi}{2\sqrt{2}}$.

23.1. Obtain the character table of the group of all symmetry operations of a square.

OR

2. State and Prove Schur's lemma.

24.1. Using Laplace transform solve $X' + 4X = t$, with $X(0) = 0$.

OR

2. Solve Poisson's equation by constructing the Green's function for it.

(12 x 3 = 36)