Reg. No .... Name **20P1004**

MSc DEGREE END SEMESTER EXAMINATION - OCT. 2020 : FEBRUARY 2021

SEMESTER 1 : PHYSICS

COURSE : 16P1PHYT01 : MATHEMATICAL METHODS IN PHYSICS - I

(For Regular - 2020 Admission and Supplementary - 2016/2017/2018/2019 Admissions)

Time : Three Hours Max. Marks: 75

PART A
Answer All (1 mark each)

1. If $\vec{r}$ is the position vector, then $\nabla \times \vec{r}$
   (a) 0 (b) 3 (c) $r^2\vec{r}$ d) $r^{3/2}$
2. The number of linearly independent eigen vectors of $\begin{bmatrix} 2 & 1 \\ 0 & 2 \end{bmatrix}$ is
   (a) 0 (b) 1 (c) 2 (d) infinite
3. Let A, B be two complex $n \times n$ matrices that are Hermitian and $C_1 = A + B$, $C_2 = iA(2+3i)$ and $C_3 = AB$. Then, among C_1, C_2, C_3, which is/are Hermitian?
   (a) Only C_1 (b) Only C_2 (c) Only C_3 (d) All of them
4. The value of δ^{12}_{21} is
   (a) 1 (b) 0 (c) -1 (d) 2
5. Which of the following is true for Dirac delta function?
   (a) $x\delta(x) = 0$ (b) $x\delta(x) = 1$ (c) $x\delta(x) = x$ (d) $x\delta(x) = \infty$

(1 x 5 = 5)

PART B
Answer any 7 (2 marks each)

6. Express position and velocity of a particle in cylindrical coordinates.
7. Show that curl of a vector is always sinusoidal in nature.
8. Explain Poisson's distribution with an example.
9. Show that product of two unitary matrices are also unitary.
10. Prove that the eigen values of a real symmetric matrix are all real.
11. What are fundamental tensors?
12. Find differential length ds^2 in cylindrical coordinates.
13. Define Dirac delta function. State one situation where it finds application.
14. Write any two definitions of Gamma function.
15. Write any two transformation equations of Beta function.

(2 x 7 = 14)

PART C
Answer any 4 (5 marks each)

16. State and prove Schwarz's inequality relation.
17. Explain elementary probability theory. Find out the probability of getting a sum 9 from two throws of a dice?
18. Define the direct product of a matrix. Find out the direct product of $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ and $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$.

19. What is contraction of a tensor? Show that contraction produces a tensor with a rank reduced by 2.
20. Determine the conjugate metric tensor in cylindrical coordinates
21. Express sin(x) in terms of $J_n(x)$.

(5 x 4 = 20)

PART D

Answer any 3 (12 marks each)

22.1. Obtain a set of four orthonormal vectors from the following linearly independent vectors (1, 1, 0, 1), (1, 0, 0, 2), (0, 1, 2, -3), (1, 1, 1, 1).

OR

2. State and prove Greens theorem. Using Green's theorem find the area of an ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1.$

23.1. Find the inverse of the given matrix using Cayley Hamilton theorem and verify it using Gauss Jordan method:

$$\begin{bmatrix} 3 & 1 & 1 \\ 1 & 3 & 2 \\ 2 & 2 & 3 \end{bmatrix}$$

OR

2. Derive the Rodrigues Formula for Legendre polynomial of order n. Hence deduce the values of $P_o(x)$, $P_1(x)$ and $P_2(x)$.

24.1. Show that y = $\int_0^{\pi} (x cos\phi d\phi)$ satisfy the equation $y'' + \frac{1}{x}y' + y = 0$ and that y's are no other than $J_n(x)$.

OR

2. Derive Rodrigues formula, generating function and any two recurrence relation of Hermite polynomials.

(12 x 3 = 36)