Reg. No………………………………………….. Name…………………………………………… **20U629**

B.Sc. DEGREE END SEMESTER EXAMINATION - MARCH 2020

SEMESTER –6: MATHEMATICS (CORE COURSE)

COURSE: 15U6CRMAT11: DISCRETE MATHEMATICS

(Common for Regular 2017 Admission & Supplementary 2016 /2015/2014 Admissions)

Time: Three Hours Max Marks: 75

PART A

Answer all questions. Each question carries 1 mark.

1. Define a graph G.
2. Find the number of edges in an acyclic graph with 19 vertices and 4 connected components.
3. Define cut vertex of a graph.
4. Draw a graph which has an Euler trail but not Eulerian.
5. Draw a graph and mark a perfect matching in it.
6. State Hall's Marriage Theorem.
7. Distinguish between plain text and cipher text.
8. State Knapsack problem.
9. Prove that if L is a lattice and if 0, u $\in$ L, then $0 \wedge a = 0$ and $u \wedge a = a$ for all $a \in L$.
10. State the absorption laws of lattice. (1 x 10 = 10)

PART B

Answer any eight questions. Each question carries 2 mark.

11. Define graph isomorphism.
12. Write the adjacency matrix of $K_{1,1}$.
13. Define vertex connectivity of a graph G. Find $\kappa(K_n)$.
14. Give an example of a graph which has a Hamiltonian path but no hamiltonian cycle.
15. Find the closure of C_4.
16. Is a maximum matching perfect matching. Justify.
17. Encrypt the message 'RETURN HOME' using the linear cipher $C \equiv P + 3 \pmod{26}$.
18. Solve the super increasing Knapsack Problem $54 = x_1 + 2x_2 + 5x_3 + 9x_4 + 18x_5 + 40x_6$.
19. Define a Poset. Give an example.
20. Prove that $a \wedge (b \wedge c) = (a \wedge b) \wedge c$. (2 x 8 = 16)

PART C

Answer any five questions. Each question carries 5 mark.

21. If u and v are any two vertices of a graph G, then prove that every u-v walk contains a u-v path.
22. Prove that an edge e in a graph G is a bridge if and only if e is not part of any cycle in G.
23. Let G be a graph with n vertices where $n \geq 2$. Then prove that G has atleast two vertices which are not cut vertices.
24. Prove that a simple graph G is Hamiltonian if and only if its closure is C(G) is Hamiltonian.
25. Write a short note on Personnel Assignment Problem.
26. Decrypt the message BS FMX KFSGR JAPWL which was produced from a vignere cipher with keyword 'YES'.
27. Prove that a mapping $f: P \rightarrow Q$ is an isomorphism if and only if f is isotone and has an isotone inverse.

(5 x 5 = 25)

PART D

Answer any two questions. Each question carries 12 mark.

28. Prove that if G is a non-empty graph with at least two vertices then G is bipartite if and only if G has no odd cycles.
29. State and Prove Whitney's theorem.
30. A user of the Knapsack Cryptosystem has a private key consisting of the super increasing sequence 3, 5, 11, 20, 41, the modulus m = 85 and multiplier a = 44.

    (a) Find the users listed public key.

    (b) With the aid of the public key encrypt the message 'HELP US'
31. (a) Prove that a Poset $(L, \leq)$ is a Lattice if and only if every nonempty finite subset of L has Sup and Inf.

    (b) Prove that in a lattice L, $a \wedge (b \vee c) \geq (a \wedge b) \vee (a \wedge c) \quad \forall\, a,b,c \in L$

(12 x 2 = 24)

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