Reg. No... Name... **20U616**

B. Sc. DEGREE END SEMESTER EXAMINATION - MARCH 2020

SEMESTER –6: MATHEMATICS (CORE COURSE)

COURSE: 15U6CRMAT10: COMPLEX ANALYSIS

(Common for Regular 2017 Admission & Supplementary 2016 /2015/2014 Admissions)

Time: Three Hours Max Marks: 75

PART A

Answer All questions. Each question carries 1 mark.

1. Define harmonic function.
2. Write the function $f(z) = e^z$ in the form $u(x, y) + i\, v(x, y)$.
3. Define hyperbolic sine function of a complex variable.
4. State Cauchy-Goursat theorem.
5. Define a simply connected domain.
6. Evaluate $\int_0^{\frac{\pi}{4}} e^{it}\, dt$.
7. Give an example of an essential singularity.
8. State Jordan's lemma.
9. Find the residue at $z = 2i$ of the function $f(z) = \frac{z^2}{z^2+4}$.
10. Write the Maclaurin series expansion of $\frac{1}{1-z}$ if $|z| < 1$. (1 x 10 = 10)

PART B

Answer any Eight questions. Each question carries 2 marks.

11. Verify Cauchy – Riemann equation for the function $f(z) = sinx\ coshy + i\ cosx\ sinhy$.
12. Show that $u = y^3 - 3x^2y$ is a harmonic function.
13. Show that $Log(i^3) \neq 3\ Log\ i$.
14. Evaluate $\int_C \frac{z^2+1}{z^2-1}\, dz$ if C is the circle of unit radius with centre at $z = 1$.
15. Evaluate $\int_C \frac{dz}{z-a}$ where C is the circle $|z - a| = r$.
16. Evaluate $\int_C \frac{z+2}{z}\, dz$ where C is the semi circle $z = 2e^{i\theta}$, $0 \leq \theta \leq \pi$.
17. Expand $\sin z$ by Taylor's series about the point $z = \frac{\pi}{2}$.
18. Prove that $\frac{1}{4z-z^2} = \frac{1}{4z} + \sum_{n=0}^{\infty} \frac{z^n}{4^{n+2}}$ when $0 < |z| < 4$.
19. What is the nature of the singularity of the function $f(z) = \left(\frac{z+1}{z^2+1}\right)^2$.
20. Find the residue of $f(z) = cot\ z$ at its singular points. (2 x 8 = 16)

PART C

Answer any Five questions. Each question carries 5 marks.

21. Show that if $f(z) = u + i\,v$ be an analytic function, then u and v are both harmonic functions.
22. Show that the function $f(z) = |z|^2$ is differentiable at the origin but nowhere else.
23. State and prove fundamental theorem of algebra.
24. Find the value of the integral of $f(z)$ around the circle $|z - i| = 2$ in the + ve sense when (i) $f(z) = \frac{1}{z^2+4}$ (ii) $f(z) = \frac{1}{(z^2+4)^2}$
25. Derive the Taylor series representation.
26. State and prove Cauchy's residue theorem.
27. Verify that that the sum of the residues of the function $f(z) = \frac{z^3}{(z-1)(z-2)(z-3)}$ at its poles and at infinity is zero. (5 x 5 = 25)

PART D

Answer any Two questions. Each question carries 12 marks.

28. Derive Cauchy Riemann equations.
29. (i) State and prove Cauchy's integral formula.
    (ii) State and prove fundamental theorem of algebra
30. (i) State Laurent's theorem.
    (ii) Find the two Laurent series expansions in powers of z of the function $f(z) = \frac{1}{z(1+z^2)}$.
31. Evaluate the integral $\int_0^\infty \frac{x^2}{x^6+1}\,dx$. (12 x 2 = 24)

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