Reg. No………………………….……………….. Name…………………………………………… **20U437**

B. Sc. DEGREE END SEMESTER EXAMINATION - MARCH 2020

SEMESTER **– 4: MATHEMATICS (COMPLEMENTARY COURSE FOR PHYSICS AND CHEMISTRY)**

COURSE: **15U4CPMAT04, FOURIER SERIES, DIFFERENTIAL EQUATIONS, NUMERICAL ANALYSIS AND ABSTRACT ALGEBRA**

(For Regular - 2018 Admission and Supplementary / Improvement 2017, 2016, 2015, 2014 Admissions)

Time: Three Hours Max. Marks: 75

PART A

*Answer **all** questions. Each question carries **1** mark.*

1. Define Fundamental period.
2. Define Fourier Series of a 2π Periodic function $f(x)$
3. Define Bessel's function of first kind of order *v*
4. Write the Newton's iteration formula for finding the square root of N.
5. Find the relative error of the number 7.6 if both of its digits are correct
6. Form the partial differential equation by eliminating the constants for $z = (x^2 + a)(y^2 + b)$
7. Write the Lagrange's Partial differential equation
8. Find solution of the differential equation $p - q = 1$
9. State the left and right cancellation laws in a group a group G with binary operation *
10. Find the order of the cyclic subgroup generated by $5 \in \mathbb{Z}_{12}$ (1 × 10 = 10)

PART B

*Answer **any eight** questions. Each question carries **2** marks.*

11. Find the half range cosine series of $f(x) = x,\ 0 < x < 1$
12. Find the power series solution of $y'' + y = 0$
13. Solve the differential equation $x^2y'' + xy' + \left(x^2 - \frac{1}{9}\right)y = 0$
14. Explain Newton-Raphson Method
15. Find a real root of the equation $x^3 - 3x - 5 = 0$ correct to three decimal places, using bisection method
16. Using Iteration Method find the root of the equation $2x = cosx + 3$ correct to two decimal places
17. Form the partial differential equation of all spheres of radius 'a' whose center's lie on the xy-plane
18. Solve the partial differential equation $p\ tanx + q\ tany = tanz$
19. If every element of a group be its own inverse, then show that the group is Abelian
20. If R is a ring with additive identity '0', then for any $a, b \in R$ Prove that i) $0a = a0 = 0$ and ii) $a(-b) = (-a)b = -(ab)$ (2 × 8 = 16)

PART C

*Answer **any five** questions. Each question carries **5** marks.*

21. Find the Fourier series of the function $f(x) = x + \pi \;\; if -\pi < x < \; \pi$ and $f(x + 2\pi) = f(x)$
22. Define Rodrigues's formula. Using Rodrigues formula find the first five Legendre Polynomials
23. Find a real root of the equation $x^3 - 9x + 1 = 0$ correct to three decimal places, using regula falsi method
24. Using Newton Raphson Method, find a root of the equation $2sinx = x$
25. Form the partial differential equation by eliminating the arbitrary function from
$$z = f(x + it) + g(x - it)$$
26. Find the general integral of the linear partial differential equation $z\,p - z\,q = z^2 + (x + y)^2$
27. Prove that set $\{a + b\sqrt{2} : a, b \in Z\}$ is a ring with respect to ordinary addition and ordinary multiplication (5 × 5 = 25)

PART D

*Answer **any two** questions. Each question carries **12** marks.*

28. Find the Fourier series of the function $f(x) = \frac{1}{2}(\pi - x) \quad 0 < x < 2\pi$, hence deduce that $1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots = \frac{\pi}{4}$
29. a) Find the real root of the equation $x^3 + x^2 - 1 = 0$ on the interval [0, 1] correct to four decimal places, using iteration method

    b) Use Newton-Raphson method to find a root of the equation $x^3 - 2x - 5 = 0$
30. a) Find the integral curves of the equations $\frac{dx}{x+z} = \frac{dy}{y} = \frac{dz}{z+y^2}$

    b) Find the general integrals of the linear partial differential equation $(y + zx)p - (x + yz)q = x^2 - y^2$
31. a) Show that the set Q^+ of all positive rational numbers forms an abelian group under the operation defined by $a * b = \frac{ab}{2}$

    b) Give the multiplication table of symmetric group of 3 elements, also show that it is not Abelian (12 × 2 = 24)

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