Reg. No………………………………………….. Name…………………………………………… **20U419**

B. Sc. DEGREE END SEMESTER EXAMINATION - MARCH 2020

SEMESTER **– 4: MATHEMATICS (CORE COURSE FOR MATHEMATICS & COMPUTER APPLICATIONS)**

COURSE: **15U4CRMAT04-15U4CRCMT04, VECTOR CALCULUS, THEORY OF EQUATIONS AND NUMERICAL METHODS**

(For Regular - 2018 Admission and Supplementary / Improvement 2017, 2016, 2015 Admissions)

Time: Three Hours Max. Marks: 75

PART A

*Answer **all** questions. Each question carries **1** mark.*

1. Find the angle between the straight lines x-y =1 and x-2y = -1.
2. Find the parametric equation of the line passing through the origin parallel to i+2j+k.
3. Evaluate $\int_{\frac{-\pi}{4}}^{\frac{\pi}{4}}\left(\sin t\vec{i}+(1+\cos t)\vec{j}+\sec^2 t\vec{k}\right)dt$.
4. Integrate $f(x,y,z)=(x-3y^2+z)$ over the line segment C joining the origin and the point (1,1,1).
5. Check whether $F=yz\vec{i}+xz\vec{j}+xy\vec{k}$ is conservative.
6. Find the area of the region enclosed by $r(t)=a\cos t\vec{i}+a\sin t\vec{j}, 0\le t\le 2\pi$.
7. Find the gcd of x^2+7x+6 and x^2-5x-6.
8. Form an equation whose roots are the negative of the roots of $2x^3-5x^2+7=0$.
9. Find the first approximate root x_1 using Newton Raphson formula to solve $x^3+x-1=0$ starting from $x_0=1$.
10. Write the formula to find the approximate solution of an equation using Regula Falsi method.

(1 x 10 = 10)

PART B

*Answer **any eight** questions. Each question carries **2** marks.*

11. Find the distance from S(1, 1, 5) to the line $x=1+t, y=3-t, z=2t$.
12. Solve the initial value problem for r as a vector function of t. $\frac{d^2r}{dt^2}=-32\vec{k}$,

    $r(0)=0, \frac{dr}{dt/_{t=0}}=8\vec{i}+8\vec{j}$.
13. Find the unit tangent vector to the curve $r(t)=\cos^3 t\vec{j}+\sin^3 t\vec{k}, 0\le t\le\frac{\pi}{2}$.
14. If $F=\nabla f$, find the potential function f for $F=2x\vec{i}+3y\vec{j}+4z\vec{k}$.
15. Find the curl of $F=x^2z\vec{i}-2y^3z^2\vec{j}+xy^2z\vec{k}$.
16. Evaluate $\iint_S (7xi-zk)\cdot n d\sigma$ over a sphere $S: x^2+y^2+z^2=4$ using divergence theorem.

17. Solve $2x^3 - 9x^2 - 27x + 54 = 0$ given that its roots are in geometric progression.
18. Form an equation whose roots are 2 times the roots of $2x^3 - 5x^2 + 7 = 0$.
19. Find an approximate root of the equation $2x = \cos x + 3$ by iteration method in four steps starting from $x_0 = \frac{\pi}{2}$.
20. Calculate $\sqrt{7}$ by Newton's iteration starting from $x_0 = 2$ in three steps. (2 x 8 = 16)

PART C

*Answer **any five** questions. Each question carries **5** marks.*

21. Find the radius of the osculating circle to the curve $y = x^2$ at the origin**.**
22. Find the torsion τ for the helix $r(t) = a\cos t\vec{i} + a\sin t\vec{j} + bt\vec{k}$.
23. Show that $2xdx + 2ydy + 2zdz$ is exact and evaluate $\int_{(0,0,0)}^{(2,3,-6)} 2xdx + 2ydy + 2zdz$.
24. Find the area of the cap cut from the hemisphere $x^2 + y^2 + z^2 = 2,\ z \geq 0$ by the cylinder $x^2 + y^2 = 1$
25. Remove the second term from the equation $x^3 - 6x^2 + 4x - 7 = 0$.
26. Solve the equation $6x^4 + 5x^3 - 38x^2 + 5x + 6 = 0$.
27. Find a root of $x^3 + x^2 - 1 = 0$ by fixed point iteration method on $[0,1]$ in four steps. (5 x 5 = 25)

PART D

*Answer **any two** questions. Each question carries **12** marks.*

28. Find the
    a) unit normal to the surface $f(x, y, z) = x^3 - xy^2 - z$ at P_0(1,1,0).
    b) Find the derivative of f in the direction of $2\vec{i} - 3\vec{j} + 6\vec{k}$ at P_0.
    c) In what direction does f change most rapidly at P_0 and what are the rates of change in these directions.
29. Find the outward flux of $F = yz\vec{i} + x\vec{j} - z^2\vec{k}$ through the parabolic cylinder $y = x^2$, $0 \leq x \leq 1, 0 \leq z \leq 4$.
30. Solve the equation $x^4 - 3x^2 - 6x - 2 = 0$ by Ferrari's method.
31. a) Find an approximate root of $x^3 - x - 4 = 0$ lying between $a = 1$ and $b = 2$ using bisection method up to 3 steps.
    b) Find an approximate solution to the equation $xe^x = 2$ lying between 0 and 1 using regula falsi method in 3 steps. (12 x 2 = 24)

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