Reg. No………………………………………….. Name…………………………………………………… **20U242**

B.Sc. DEGREE END SEMESTER EXAMINATION - MARCH 2020

SEMESTER - 2: MATHEMATICS (COMPLEMENTARY COURSE FOR PHYSICS & CHEMISTRY)

COURSE CODE: 19U2CPMAT2: CALCULUS II AND NUMERICAL ANALYSIS

(For Regular - 2019 Admission)

Time: Three Hours Max. Marks: 75

PART A

Answer any Ten questions. Each question carries 2 marks

1. Find the velocity and acceleration of the particle which moves along the curve $x = 2Sin\ 3t, y = 2\cos 3\,t, z = 8t$ at $t = \frac{\pi}{12}$.
2. If $\phi = x^3yz^3$ find $\nabla\emptyset$ at (1,1,1)
3. If $\bar{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$ find div $\bar{r}$ and curl $\bar{r}$.
4. Given the vector field $\bar{F} = xz\,\hat{\imath} + yz\,\hat{\jmath} + z^2\,\hat{k}$. Evaluate $\int_C \bar{F}.d\bar{r}$ where C is the straight line from the point (0, 0, 0) to (1,1,1).
5. Evaluate $\oint_C \bar{F}.d\bar{r}$ by Stoke's Theorem, where $\bar{F} = y^2\,\hat{\imath} + x^2\hat{\jmath} - (x+z)\hat{k}$ and C is the boundary of the triangle with vertices at (0,0,0), (1,0,0) and (1,1,0).
6. Find $\iint_S \bar{F}\,.\,\hat{n}\,dS$ where $\bar{F} = (2x+3z)\hat{\imath} - (xz+y)\hat{\jmath} + (y^2+2z)\hat{k}$ and S is the surface of the sphere having center at (3,-1, 2) and radius 3.
7. Given that

$x:$	1	2	3	4	5
$y:$	2	5	10	17	26

Find the value of $\nabla^2 y_5$.

8. Show that $(1+\Delta)(1-\nabla) = 1$.
9. Use trapezoidal rule to evaluate $\int_4^8 \frac{dx}{x}$, using four equal subintervals.
10. Find the numerical value of the first derivative of the function $f(x)$defined as

$x:$	0.1	0.2	0.3	0.4
$y:$	1.10517	1.2214	1.34986	1.49182

11. Find by iteration method, a real root of $2x - log_{10}x = 7$.
12. Find a root of $3x - 1 = \cos x$ correct to three decimal places using Newton Raphson's method.

$(2 \times 10 = 20)$

PART B

Answer any Five questions. Each question carries 5 marks.

13. Prove that $\nabla r^n = n\, r^{\,n-2} \bar{r}$ where $\vec{r} = x\hat{\imath} + y\,\hat{\jmath} + z\,\hat{k}$ and $r = |\bar{r}|$
14. Prove that $f(x)\,\vec{r}$ is irrotational.
15. Evaluate $\iint_S \bar{A}\,.\,\hat{n}\,ds$ where $\vec{A} = z\hat{\imath} + x\,\hat{\jmath} - 3yz\,\hat{k}$ and S is the surface of the cylinder $x^2 + y^2 = 16$ included in the first octant between $z = 0$ and $z = 5$
16. If $\bar{F} = z\hat{\imath} + by\,\hat{\jmath} + cz\,\hat{k}$ where a, b, c are constants, show that $\iint_S \bar{F}\,.\,d\bar{S} = \frac{4}{3}\pi(a + b + c)$ where S is the surface of the unit sphere.
17. If 'D' stands for the differential operator $\frac{d}{dx}$, prove that $D = \frac{1}{h}[\Delta - \frac{1}{2}\Delta^2 + \frac{1}{3}\Delta^3 - \cdots]$.
18. Using the following values apply Lagrange's interpolation formula to find the value of $f(4)$

x :	0	2	3	6
$f(x)$:	-4	2	14	158

19. Compute the real root of the equation $xlog_{10}^{x} - 1.2 = 0$ correct to three decimal places using Newton – Raphson method.
20. Find a root of the following equation correct to three decimal places using Honer's method $x^3 + 3x^2 - 12x - 11 = 0.$

$(5 \times 5 = 25)$

PART C

Answer any Three questions. Each question carries 10 marks.

21. (a) If $\bar{A} = 3xz^2\hat{\imath} - yz\,\hat{\jmath} + (x + 2z)\,\hat{k}$ find $Curl(Curl\bar{A})$.

    (b) If $\vec{r} = x\hat{\imath} + y\,\hat{\jmath} + z\,\hat{k}$ prove that $\frac{\vec{r}}{r^3}$ is solenoidal.
22. Verify divergence theorem for $\bar{F} = 4x\hat{\imath} - 2y^2\,\hat{\jmath} + z^2\,\hat{k}$ taken over the region bounded by the cylinder $x^2 + y^2 = 4, z = 0, z = 3$.
23. The following table gives corresponding values of x and y. Prepare a forward difference table and express y as a function of x. Also obtain y when $x = 2.5$.

x :	0	1	2	3	4
$f(x)$:	7	10	13	22	43

24. Find all root of the equation $x^4 - 3x + 1 = 0$ by Graeffe's method.

$(10 \times 3 = 30)$

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