

B.Sc. DEGREE END SEMESTER EXAMINATION - MARCH 2020**SEMESTER –2: MATHEMATICS (CORE COURSE)****COURSE: 19U2CRMAT2: ADVANCED CALCULUS AND TRIGONOMETRY***(For Regular - 2019 Admission)*

Time: Three Hours

Max Marks: 75

Part-A**(Answer any 10 questions. Each questions carries 2 marks)**

1. Find the n^{th} derivative of $\cos(ax + b)$.
2. Find the Maclaurin series expansion of $\sinh x$.
3. Define curvature and radius of curvature of a curve $y = f(x)$ at any point P.
4. Find the envelop of the family of lines $y = mx + a\sqrt{1 + m^2}$ where m is the parameter.
5. Find the length of the curve $y = \log \sec x$ between the points given by $x = 0$ and $x = \frac{\pi}{4}$.
6. Find the total arc length of the hypercycloid $x = a\cos^3\theta$, $y = a\sin^3\theta$.
7. Find the area of the region enclosed by the cardioid $r = a(1 + \cos \theta)$.
8. Find by double integration, the volume of the solid generated by revolving the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ about the y axis.
9. Change the order of integration in $\int_{-a}^a \int_0^{\sqrt{a^2-y^2}} f(x, y) dx dy$.
10. Evaluate $\int_0^{\frac{\pi}{2}} \int_0^{a \cos \theta} r^4 dr d\theta$.
11. Separate into real and imaginary parts the expression $\tan^{-1}(x + iy)$.
12. Prove that $\cosh(x + iy) = \cosh x \cos y + i \sinh x \sin y$.

Part-B

(Answer any 5 questions. Each questions carries 5 marks)

13. State and prove Leibnitz theorem.
14. If $y = \cos(m\sin^{-1}x)$, prove that $(1 - x^2)y_{n+2} - (2n + 1)xy_{n+1} + (m^2 - n^2)y_n = 0$.
15. Prove that the area of a loop of the curve $r = a \sin 3\theta$ is $\frac{1}{12}\pi a^2$.
16. Find the length of the arc of the curve $x = t^2$, $y = t^3$ between $t = 0$ and $t = 1$.
17. Find the sum of the following series to infinity: $\cos x \sin x + \frac{1}{2!}\cos^2 x \sin 2x + \frac{1}{3!}\cos^3 x \sin 3x + \dots$
18. Factorize $x^7 - 1$ into real factors.
19. Evaluate $\int_0^1 \int_0^{1-x} \int_0^{x+y} e^z dz dy dx$.
20. If x is real, show that $\sinh^{-1} x = \log(x + \sqrt{x^2 + 1})$.

Part-C

(Answer any 3 questions. Each questions carries 10 marks)

21. a. Find the coordinates of the centre of curvature at the point $x = at^2$, $y = 2at$ on the parabola $y^2 = 4ax$ and hence find its evolute.
b. Prove that the evolute of the cycloid $x = a(\theta - \sin \theta)$, $y = a(1 - \cos \theta)$ is another cycloid.
22. Find the volume of the solid enclosed by the sphere $x^2 + y^2 + z^2 = a^2$.
23. a. Express $\sin^8 \theta$ in a series of cosines of multiples of θ .
b. Expand $\cos 7\theta$ in descending powers of $\cos \theta$.
24. Change the order of integration in $\int_0^a \int_{\frac{x^2}{a}}^{2a-x} xy \, dx dy$ and hence evaluate the same.