20P4047

Reg. No .... Name ....

MSc DEGREE END SEMESTER EXAMINATION - MARCH 2020

SEMESTER 4 : MATHEMATICS

COURSE : **16P4MATT20EL : NUMERICAL ANALYSIS**

(For Regular - 2018Admission and Supplementary - 2017, 2016 Admission)

Time : Three Hours Max. Marks: 75

Section A

Answer All the Following (1.5 marks each)

1. Write the expression of absolute error E_A in the quotient a/b.
2. Define round off error and truncation error.
3. State Taylor's series for a function of several variables $x_1, x_2, x_3, \dots x_n$
4. Obtain the total number of arithmetic operations in Gauss elimination method.
5. Define condition number of a matrix with example.
6. Prove that $\nabla = 1 - E^{-1}$
7. Prove that $\delta = E^{1/2} - E^{-1/2}$
8. Define shift operator and averaging operator.
9. Express lagrange polynomial of degree one passing through two points.
10. Solve the equation $y' = x + y^2$, subject to the condition y = 1 when x = 0.

(1.5 x 10 = 15)

Section B

Answer any 4 (5 marks each)

11. Let $x = \epsilon$ be a root of f(x)=0 and let I be the interval containing the point $x = \epsilon$. Let $\phi(x)$ and $\phi'(x)$ be continuous in I where $x = \phi(x)$ is equivalent to $f(x) = 0$. Then if $|\phi'(x)| < 1$ for all x in I , the sequence of approximatons $x_0, x_1, , \dots x_n$ x_0 defined by $x_{n+1} = \phi(x_n)$ converges to the root ϵ, provided the initial approximation is chosen in I.
12. Write the Taylor's series expansion of f(x) = cos x at $x = \pi/3$ in terms of f(x). Compute the approximations from zero - order to the fifth order and also state the absolute error in each case.
13. Briefly explain the method of LU Decomposition.
14. Find the third degree Hermite polynomial passing through the points $(x_i, y_i, y_i'); i = 0, 1.$
15. Evaluate $\int_0^1 \frac{1}{1+x} dx$ using (a) Simpson's rule, taking $h = 0.25$ (b) Trapezoidal rule, taking $h = 0.5.$
16. Use Euler's method to solve $y' = -2y$ with the condition y(0)=1.Find y(0.1), y(0.2) and y(0.3) by taking h =0.1.

(5 x 4 = 20)

Section C
Answer any 4 (10 marks each)

17.1. Describe the algorithm to solve Regula - Falsi method and hence solve the equation $xe^x = 1$ whose roots lie between 0 and 1.

OR

2. Solve the system of equation 10x-2y-z-t=3, -2x+10y-z-t=15, -x-y+10z-2t=27, -x-y-2z+10t=-9 using Gauss-Seidel method and Jacobi's method.

18.1. Decompose the matrix $A = \begin{bmatrix} 5 & -2 & 1 \\ 7 & 1 & -5 \\ 3 & 7 & 4 \end{bmatrix}$ in to the form LU.

OR

2. Derive Newton's general interpolation formula with divided differences.

19.1. The population of a town in decennial censur were given below: Estimate the population for the year 1955

Year	1921	1931	1941	1951	1961
Population (in 1000's)	46	66	81	93	101

OR

2. Given $y' = y - x$ where y(0)=2, find y(0.1) and y(0.2) with h = 0.05.

20.1. Solve $y' = 2 + \sqrt{(xy)}$, y(1)=1, to find the value of y(2) in steps of 0.1 using Euler's modified method.

OR

2. Using Milne's method, solve the differential equation $(1 + x)y' + y = 0$,with y(0)=2, for x = 1.5 to 2.5. Obtain the starting values by using the fourth order Runge-Kutta method with h=0.5

(10 x 4 = 40)