20P4028

Reg. No .... Name ....

MSc DEGREE END SEMESTER EXAMINATION - MARCH 2020

SEMESTER 4 : MATHEMATICS

COURSE : **16P4MATT18EL : COMBINATORICS**

(For Regular - 2018 Admission and Supplementary - 2017, 2016 Admissions)

Time : Three Hours Max. Marks: 75

Section A

Answer All the Following (1.5 marks each)

1. What is the relation between Q_r^n and P_r^n ?
2. Prove by a combinatorial argument that the following number is always an integer for each $n \epsilon N$: $\frac{(n!)!}{(n!)^(n-1)!}$.
3. Show that (4n)! is a multiple of 2^{3n} . 3^n , for each natural number n.
4. State the Generalized Pigeonhole Principle.
5. Prove that among any group of 13 people, there must be at least 2 whose birthdays are in the same month.?
6. Define Ramsey number.
7. Derive the formula for $|A \cup B \cup C|$
8. Let S = {1,2, ... ,100}. Find the number of integers in S which are divisible by 2?.
9. Find the generating function for the sequence (c_r), where,$c_r = \begin{cases} 0 \quad if \;\; 0 \leq m-1 \\ a_{r-m} \quad if \quad r \geq m \end{cases}$
10. Let a_r be the number of partitions of an integer r into parts of sizes 1, 2 or 3. Find the generating function for (a_r)

(1.5 x 10 = 15)

Section B

Answer any 4 (5 marks each)

11. Find the number of postive divisors of 600, inclusive of 1 and 600 itself?
12. Find the number of common positive divisors of 10^{40} and 20^{30}
13. Prove that for all integers $p, q \leq 2$, $R(p, q) \leq R(p-1, q) + R(p, q-1)$.
14. Show that $\sum_{k=0}^{q} (-1)^k \omega(k)$
15. Explain and find the formula for Stirling number of the 2nd kind?
16. In how many ways can 4 of the letters from PAPAYA be arranged using generating functions

(5 x 4 = 20)

Section C

Answer any 4 (10 marks each)

17.1. a) If there must be at least one person in each table, in how many ways can 6 people be seated?
(i) around two tables?
(ii) around three tables?
(We assume that the tables are indistinguishable.)
b) Show that s(r, n) = s(r - 1, n - 1) + (r - 1)s(r - 1, n)

OR

2. a) In a group of 15 students, 5 of them are female. If exactly 3 female students are to be selected, in how many ways can 9 students be chosen from the group
(i) to form a committee?
(ii) to take up 9 different posts in a committee?
b) Ten chairs have been arranged in a row. Seven students are to be seated in seven of them so that no two students share a common chair. Find the number of ways this can be done if no two empty chairs are adjacent.
c) Let $r \epsilon N$, such that
$$\frac{1}{\binom{9}{r}} - \frac{1}{\binom{10}{r}} = \frac{11}{6\binom{11}{r}}.$$
Find the value of r ?

18.1. Explain the Bounds for Ramsey numbers

OR

2. a) Show that for all intergers $p, q \geq 2$, if R(p-1,q) and R(p,q-1) are even , then R(p,q) $\leq$ R(p-1,q) + R(p,q-1) - 1.
b) Six points are in general position in space (no three in a line, no four in a plane). The fifteen line segments joining them in pairs are drawn, and then painted with some segments red and the rest blue. Prove that some triangle has all its sides the same colour.

19.1. a) Find the number of nonnegative integer solutions to the equation $x_1 + x_2 + x_3 = 15$; using GPIE $x_1 \leq 5,\ x_2 \leq 6,\ x_3 \leq 7.$
b) Prove that for any $n \epsilon N,\ Lt_{x \to \infty} \left(\frac{D_n}{n!}\right) = e^{-1} \simeq 0.367$

OR

2. Explain the problem of Tower of Hanoi with diagrams

20.1. a) Solve the recurrence relation $a_n - 3a_{n-1} = 2 - 2n^2$); Given that $a_0 = 3$.
b) Prove that for each $n \epsilon N$, the number of partitions of n into parts each of which appears at most twice, is equal to the number of partitions of n into parts the sizes of which are not divisible by 3.

OR

2. a) Show that the generating function for the number of ways to select r objects from 3 distinct objects is $(1+x)^3$.
b) Express the generating function for each of the following sequences (c_r) in closed form (i.e., a form not involving any series):
i) $c_r = 3r + 5\ for\ each\ r\ \epsilon\ N^*$;.
$ii) c_r = r^2\ for\ each\ r\ \epsilon\ N^*$;

(10 x 4 = 40)