20P4016

Reg. No .... Name ....

MSc DEGREE END SEMESTER EXAMINATION - MARCH 2020

SEMESTER 4 : MATHEMATICS

COURSE : **16P4MATT17EL : MULTIVARIATE CALCULUS AND INTEGRAL TRANSFORMS**

(For Regular - 2018 Admission and Supplementary - 2017, 2016 Admissions)

Time : Three Hours Max. Marks: 75

Section A
Answer any 10 (1.5 marks each)

1. State Convolution theorem for Fourier transform.
2. Find the Laplace transform of $f(x) = \cos at$
3. Find the Laplace transform of $sinhat$
4. Explain the term Jacobian matrix
5. Let f be a function with values in $\mathbb{R}^m$ which is differentiable at a point c in $\mathbb{R}^n$ with total derivative $f'(c)$. Show that $||f'(c)(v)|| \leq M||v||$, where $M = \sum_{k=1}^{m} ||\nabla f_k(c)||$.
6. If $f = u + iv$ is a complex-valued function with a derivative at a point z in C, then $J_f(z) = |f'(z)|^2$.
7. State the inverse function theorem
8. Define Jacobian determinant
9. State Stoke's theorem.
10. Define Derivative of a 0-form and a k-forms.

(1.5 x 10 = 15)

Section B
Answer any 4 (5 marks each)

11. Derive the exponential form of Fourier Integral Theorem
12. Show that $\int_0^\infty \frac{\cos ax}{b^2+x^2} dx = \frac{\pi}{2b} e^{-|a|b}$, if $b > 0$.
13. Find the directional derivative of the function $f = x^2 - y^2 + 2z^2$ at the point $P(1,2,3)$ in the direction of the line PQ where Q is the point $5,0,4$
14. State and prove sufficient condition for a mapping to carry open sets onto open sets.
15. Show by an example that $D_{1,2}f(x,y)$ is not necessarily the same as $D_{2,1}f(x,y)$.
16. Suppose T is a $1-1$ $\zeta^{\prime}-$ mapping of an open set $E \subset R^k$ into R^k such that $J_T(x) \neq 0$ $\forall x \in E$. If f is a continuous function onR^k whose support is compactand lies in $T(E)$, then prove that $\int_{R^k} f(y)dy = \int_{R^k} f(T(x))|J_T(x)|dx$

(5 x 4 = 20)

Section C
Answer any 4 (10 marks each)

17.1. State and prove Weisrstrass Approximation theorem

OR

2. (i) Prove that $\frac{\Gamma(p)\Gamma(p)}{\Gamma(2p)} = 2\int_0^{\frac{1}{2}} x^{p-1}(1-x)^{p-1}dx$.
(ii) Make a suitable change of variable in (i) to obtain $\Gamma(2p)\Gamma\left({}^1/_2\right) = 2^{2p-1}\Gamma(p)\Gamma\left(p+\frac{1}{2}\right)$.

18.1. Assume that g is differentiable at a, with total derivative $g'(a)$. Let $b = g(a)$and assume that f is differentiable at b, with total derivative$f'(b)$. Then prove that the composite function$h = fog$ is differentiable at a, and the total derivative $h'(a)$ is given by $h'(a) = f'(b)og'(a)$, the composition of the linear functions $f'(b)$ and $g'(a)$.

OR

2. a) Show that $m(SoT) = m(S)m(T)m(SoT) = m(S)m(T)$.
b) Compute the gradient vector $\nabla f(x, y, z)$ at the point $1, 3, 5)$ of the function
$$f(x, y, z) = \frac{x^3y^5z^5}{1 + x^4 + y^3 + z^2}$$

19.1. State and prove a sufficient condition for differentiability

OR

2. a)In a plane triangle ABC, find the maximum value of $cosAcosBcosC$
b)Show that the rectangular solid of maximum volume that can be inscribed in a given sphere is a cube.

20.1. Suppose F is a $\zeta^{|}-$ mapping of an open set $E \subset R^n$ into $R^n .o \in E. F(0) = 0$ and $F^{|}(0)$ is invertible. Then prove that there is a neighbourhood of 0 in R^n in which a representation $F(x) = B_1 \ldots\ldots\ldots B_n - 1G_n o \ldots\ldots oG_1(x)$ is valid

OR

2. Prove that if ω and λ are k- and m- forms respectively of class C' in E, then $d(\omega \wedge \lambda) = (d\omega) \wedge \lambda + (-1)^k \omega \wedge d\lambda$ and if ω is of C'' in E, then $d^2\omega = 0$.

(10 x 4 = 40)