20P2055

Reg. No .... Name ....

M. Sc DEGREE END SEMESTER EXAMINATION - MARCH 2020

SEMESTER 2 : MATHEMATICS

COURSE : **16P2MATT10 : REAL ANALYSIS**

(For Regular - 2019 Admission & Supplementary 2018/2017/2016 Admissions)

Time : Three Hours Max. Marks: 75

Section A

Answer All the Following (1.5 marks each)

1. Show that a polynomial is always a function of bounded variation on every compact interval.
2. Prove that the set of discontinuities of a monotone function is countable.
3. If f is continuous on [a,b], then prove that $f \in \mathscr{R}(\alpha)$.
4. If f is Riemann integrable, then f is of bounded variation, Justify.
5. If $a < S < b$, f is bounded on $[a, b]$, f is continuous at S, and $\alpha(x) = I(x - S)$, prove that $\int\limits_a^b f d\alpha = f(S)$.
6. Discuss the uniform convergence of the sequence of functions $\{f_n(x)\}$, where $f_n(x) = \frac{x}{n}$, $x \in \mathbb{R}$.
7. State Weierstrass M-test.
8. Show by an example that the limit of a sequence of Riemann integrable functions need not be Riemann integrable.
9. Prove that $\lim\limits_{x \to \infty} x^n e^{-x} = 0$.
10. Prove that $E(it) \neq 1$, if $0 < t < 2\pi$

(1.5 x 10 = 15)

Section B

Answer any 4 (5 marks each)

11. Find the total variation of the function $f(x) = \sin 2x$ over the interval $[0, 2\pi]$.
12. Prove that $f \in \mathscr{R}(\alpha)$ if and only if $f\alpha^{'} \in \mathscr{R}$
13. Suppose α increases on $[a, b], a \leq x_0 \leq b, \alpha$ is continuous at x_0, $f(x_0) = 1$ and $f(x) = 0$ if $x \neq x_0$. Prove that $f \in R(\alpha)$ and $\int f d\alpha = 0$.
14. Prove that the sequence $\{f_n(x)\}$ is not uniformly convergent on (0,1), where $f_n(x) = x^{\frac{1}{n}}$ and discuss the uniform convergence of the same in $[k, 1],\ 0 < k < 1$.
15. If $\{f_n\}$ is a sequence of continuous functions of E and if $f_n \to f$ uniformly on E, prove that f is continuous on E.
16. Prove the algebraic completeness of the complex field.

(5 x 4 = 20)

Section C
Answer any 4 (10 marks each)

17.1. Let f and g be complex-valued functions defined as follows:

$$f(t) = e^{2\pi it} \quad \text{if} \quad t \in [0,1]; \quad g(t) = e^{2\pi it} \quad \text{if} \quad t \in [0,2]$$

a. Prove that f and g have the same graph but not equivalent.
b. Prove that the length of g is twice that of f.

OR

2. Prove that the graph of $f(x) = \begin{cases} x\cos(\pi/2x) & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$ is not rectifiable on [0,1].

18.1. Let $f \in \mathscr{R}$ on [a,b]. Prove that there is at least one anti-derivative for f on [a,b] and hence prove the fundamental theorem of calculus.

OR

2.
a. Prove that if $f \in \mathscr{R}(\alpha)$, $\int_a^b f d\alpha = \int_a^c f d\alpha + \int_c^b f d\alpha$ for $a < c < b$.
b. Evaluate $\int_1^{10} f d\alpha$ where $f(x) = [logx]$ and α is the identity function.

19.1. State the Stone-Weierstrass theorem. Prove that if f is continuous on [0,1] and $\int_a^b f(x)x^n dx = 0$ for $(n = 0, 1, 2 \ldots)$, then $f(x) = 0$ on [0,1].

OR

2. If $\{f_n\}$ and $\{g_n\}$ converge uniformly on a set E, Prove that $\{f_n + g_n\}$ converges uniformly on E. Construct sequences $\{f_n\}$, $\{g_n\}$ which converge uniformly on some set E, but such that $\{f_n g_n\}$ does not converge uniformly on E.

20.1. Introduce trigonometric functions using exponential series and hence derive any two properties of them.

OR

2.
a. If $E(z) = \sum_{n=0}^{\infty} \frac{z^n}{n!}$, prove that $E(x) = e^x \ \forall x \in \mathbb{R}$
b. If z is a complex number with $|z| = 1$, prove that there is a unique $t \in [0, 2\pi)$ such that $E(it) = z$.

(10 x 4 = 40)