20P2031

Reg. No .... Name ....

M. Sc DEGREE END SEMESTER EXAMINATION - MARCH 2020

SEMESTER 2 : MATHEMATICS

COURSE : **16P2MATT08 : ADVANCED COMPLEX ANALYSIS**

(For Regular - 2019 Admission & Supplementary 2018/2017/2016 Admissions)

Time : Three Hours Max. Marks: 75

Section A

Answer All the Following (1.5 marks each)

1. Find the radius of convergence of $\sum n^P z^n$
2. Show that $\pi_{n=2}^{\infty}(1-\frac{1}{n^2})=\frac{1}{2}$
3. State Legender's duplication formula.
4. Define a Reimann's zeta function.
5. State Hadamard's theorem
6. State Poisson's Jensen's formula
7. State Riemann mapping theorem.
8. Define harmonic function. Give an example
9. Show that for any non-constant function $f(z)$, the points of period module M are isolated.
10. Define weierstrass $\mathcal{P}$- function.

(1.5 x 10 = 15)

Section B

Answer any 4 (5 marks each)

11. Prove that `A sequence is convergent if and only if it is a cauchy sequence.
12. Prove that the necessary and sufficient condition for the absolute convergence of the product $\pi_{n=1}^{\infty}(1+a_n)$ and the convergence of the series $\sum\limits_{n=1}^{\infty}|a_n|$.
13. Prove that for $\sigma = ResS > 1$, $\frac{1}{\mathcal{C}(S)} = \pi_1^{\infty}(1-P_n^{-S})$ where P_n's are primes
14. Prove that there are infinitily many primes
15. Suppose that the boundary of a simply connected region Ω contains a line segment γ as one sides free boundary arc. Prove that the function $f(z)$ which maps Ω onto the unit disc can be extended to a function which is analytic and one-one on $\Omega \cup \gamma$. Also image of γ is an arc γ' on the unit circle.
16. Show that $\frac{\mathcal{P}'(z)}{\mathcal{P}(z)-\mathcal{P}(u)} = \zeta(z-u)+\zeta(z+u)-2\zeta(z)$.

(5 x 4 = 20)

Section C
Answer any 4 (10 marks each)

17.1. Prove the if $f(z)$ is analytic in the region Ω containing z_0 then the representation,

$$f(z) = f(z_0) + \frac{f'(z_0)}{1!}(z - z_0) + f''(z_0)\frac{(z - z_0)^2}{2!} + \cdots + f^{n-1}(z_0)\frac{(z - z_0)^{n-1}}{(n-1)!} + \cdots$$

is valid in the largest open disc with the center z_0 contained in Ω.

OR

2. Derive Legendre's Duplication formula

18.1. Let $\{b_r\}$ be a sequence of complex numbers with limit r $\rightarrow \infty b_r = \infty$ and let $P_r(\xi)$ be polynomials without constant term. Then Prove that there are functions which are meromorphic in the whole plane with poles at the points b_r and the corresponding singular parts $P_r\left(\frac{1}{z-b_r}\right)$. Moreover Prove that the most general meromorphic f_n of this band can be written in the form $f(z) = \sum\left[P_r\left(\frac{1}{z-b_r}\right) - P_r(z)\right] + g(z)$ where $b_r(z)$ are suitably choosen polynomials and $g(z)$ is analytic in the whole plane.

OR

2. a. Prove that $\zeta(S) = \frac{-\lceil 1-S}{2\pi i}\int_C \frac{(-z)^{S-1}}{e^z-1}dz$, where $(-z)^{S-1}$ is defined on the complement of the positive real axis as $e^{(S-1)\log(-z)}$, $-\pi < Im\log(-z) < \pi$

b. Prove that zeta function can be extended to a meromorphic function in the whole complex plane whose only pole is a simple pole at $S = 1$ with residue 1.

19.1. State Harnack's Principle by proving the corresponding Harnack's inequality.

OR

2. a. Show that an elliptic function without poles is a constant.

b. Prove that the sum of the residues of an elliptic function is zero.

20.1. a. Show that a non-constant elliptic function has equal number of zeroes and poles.

b. Show that the zeroes $a_1, a_2, \ldots, a_n$ and the poles $b_1, b_2, \ldots, b_n$ of an elliptic function satisfy $a_1 + \cdots + a_n \equiv b_1 + \cdots + b_n \pmod{m}$.

OR

2. Resume the first order differential equation for $\omega = P(z)$.

(10 x 4 = 40)