20P2018

Reg. No .... Name ....

M. Sc DEGREE END SEMESTER EXAMINATION - MARCH 2020

SEMESTER 2 : MATHEMATICS

COURSE : **16P2MATT07 : ADVANCED TOPOLOGY**

(For Regular - 2019 Admission & Supplementary 2018/2017/2016 Admissions)

Time : Three Hours Max. Marks: 75

Section A

Answer All the Following (1.5 marks each)

1. Let A be a subset of a space X and let $f : A \to \mathbb{R}$ be continuous. Then prove that any two extensions of f to X agree on $\bar{A}$.
2. State Tietze characterization of normality.
3. Prove that T_1-axiom is a productive property.
4. If a space is embeddable in the Hilbert cube, prove that it is second countable and T_3.
5. Define σ- locally finite family in a topological space.
6. Define a filter on a set X.
7. Define a filter associated with a net S in X.
8. If a space X is Hausdorff, prove that no filter on X can converge to more than one point in it.
9. In a second countable space prove that countable compactness implies compactness
10. Is every compact space, locally compact?. Justify.

(1.5 x 10 = 15)

Section B

Answer any 4 (5 marks each)

11. Prove that there exists no countable,connected, T_3-space.
12. Define product topology on πX_i. Let C_i be a closed subset of X_i for $i \in I$. Prove that πC_i is a closed subset of πX_i w. r. t. the product topology.
13. Prove that a topological space is completely regular iff the family of all continuous real-functions on it distinguishes points from closed sets.
14. If a net converges to a point, prove that any subnet will also converge to that point.
15. Prove that a space X is compact iff every filter on X has a cluster point in X.
16. Define countably compact space and prove that countable compactness is preserved under a continuous function.

(5 x 4 = 20)

Section C

Answer any 4 (10 marks each)

17.1. Prove that a topological product is T_0, T_1, T_2 or regular iff each co-ordinate space has the corresponding property

OR

2. Prove that a product of topological spaces is Tychonoff iff each co-ordinate space is so.

18.1. State and prove Urysohn Embedding Theorem.

OR

2. Prove that every trivial space is a Pseudo-metric space and every Pseudo-metric space is completely regular. Also prove that a space is completely regular iff it can be embedded into a product of Pseudo-metric spaces.

19.1. Let $\mathcal{F}$ be a filter in a space X and S be the associated net in X. Let $x \in X$. Prove that

a. $\mathcal{F}$ converges to x as a filter iff S converges to x as a net
b. x is a cluster point of the filter $\mathcal{F}$ iff it is a cluster point of the net S.

OR

2. Prove that the filter associated with a universal net is an ultra filter and the net associated with an ultra filter is a universal net.

20.1. Let (X^+, τ^+) be one point compactification of the space (X, τ). Prove that (i) τ^+/X is the topology τ on X (ii) The space (X^+, τ^+) is compact (iii) X is dense in X^+ iff X is not compact.

OR

2. Prove that the one-point compactification of a space is Hausdorff iff the space is locally compact and Hausdorff.

(10 x 4 = 40)