# B. Sc. DEGREE END SEMESTER EXAMINATION - MARCH 2019 <br> SEMESTER - 6: MATHEMATICS (CORE COURSE) <br> COURSE: 15U6CRMAT13: OPERATIONS RESEARCH 

(Common for Regular - 2016 Admission / Supplementary-Improvement 2015/2014 admissions)
Time: Three Hours
Max. Marks: 75

## SECTION A <br> Answer all questions

1. Define convex Hull of a set.
2. What is meant by extreme point of a convex set?
3. Define norm of a vector space.
4. The optimal solution to a linear programming problem is always unique .True or False.
5. Define artificial variables.
6. Define Loop of a transportation problem
7. Define a balanced transportation problem.
8. What do you mean by queue discipline?
9. Define waiting time of a customer in the system.
10. What is meant by traffic intensity?

## SECTION B

Answer any Eight questions
11. Define subspace of a vector space with one example
12. Write the standard form of the linear programming problem (L.P.P).
13. Formulate the L.P.P

A person has option of investing Rs. 10,000 in two plans $A$ and $B$, plan $A$ guarantees a return of 50 paisa on each rupee invested after a period of 3 years and plan B guarantees that each rupee invested will an one and a half rupees after six years. How should the person invest his money to maximize his earnings on a period of 6 years, if he is not willing to invest more than $60 \%$ in B ?
14. Show that the vector $\left[\begin{array}{cc}1-2 & -2\end{array}\right]^{1}$ and $\left[\begin{array}{ll}2-1 & 2\end{array}\right]^{1}$ are orthogonal. Find a vector orthogonal to both these vectors.
15. Write the dual of the following L.P.P
$\operatorname{Min} x_{1}+x_{2}$
Sub $2 x_{1}+x_{2} \geq 8$
$3 x_{1}+7 x_{2} \geq 21$
$x_{1}, x_{2} \geq 0$
16. Find the initial basic feasible solution of the transportation problem.

|  | $D_{1}$ | $D_{2}$ | $D_{3}$ | $D_{4}$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{O}_{1}$ | 4 | 5 | 8 | 3 | 50 |
| $\mathrm{O}_{2}$ | 5 | 4 | 3 | 2 | 30 |
| $\mathrm{O}_{3}$ | 1 | 5 | 6 | 3 | 20 |
|  | 40 | 30 | 20 | 10 |  |

17. Show that an assignment problem in a special type of linear programming problem
18. Write the different queue discipline.
19. Describe service time distribution.
20. Describe customer's behaviour in a queue.
$(2 \times 8=16)$

## SECTION C

Answer any Five questions
21. Show that vertex $S_{F}$ \{set of Basic feasible solution\} is a basic feasible solution.
22. Solve graphically.

Minimize $-4 x_{1}-5 x_{2}$
Subject to $\quad x_{1}-2 x_{2} \leq 2$

$$
\begin{gathered}
2 x_{1}+x_{2} \leq 6 \\
x_{1}+2 x_{2} \leq 5 \\
-x_{1}+x_{2} \leq 2 \\
x_{1}, x_{2} \geq 0
\end{gathered}
$$

23. Solve by dual simplex method

Minimize $2 x_{1}+3 x_{2}$
Subject to $2 x_{1}+3 x_{2} \leq 30$
$x_{1}+2 x_{2} \geq 10$
$x_{1}, x_{2} \geq 0$
24. Find the initial Basic solution of the transportation problem by VAM and find the cost.

|  | $D_{1}$ | $D_{2}$ | $D_{3}$ | $D_{4}$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $s_{1}$ | 19 | 30 | 50 | 10 | 7 |
| $s_{2}$ | 70 | 30 | 40 | 60 | 9 |
| $s_{3}$ | 40 | 8 | 70 | 20 | 18 |
|  | 5 | 8 | 7 | 14 | 34 |

25. A department has five employees with 5 jobs to be performed. The time in hours each men will take to perform each job is given with effectiveness matrix.

|  | 1 | 2 | 3 | 4 | 5 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| A | 10 | 5 | 13 | 15 | 16 |
| B | 3 | 9 | 18 | 13 | 6 |
| C | 10 | 7 | 2 | 2 | 2 |
| D | 7 | 11 | 9 | 7 | 12 |
| E | 7 | 9 | 10 | 4 | 12 |

How should the jobs be allotted one per employee so has to minimize the total man hours?
26. Show that if $X \in E_{n}, V \subseteq E_{n}$ such that $V=\left\{X\left|X=\left[x_{1}, \ldots ., x_{n}\right]^{\prime}\right| x_{1}+x_{2}+\cdots+x_{n}=0\right\}$ then $V$ is subspace of $E_{n}$.
27. If the no. of arrivals $n$, in time $t$ follows Poisson distribution. Find the distribution of the inter arrival times.
$(5 \times 5=25)$

## SECTION D <br> Answer any Two questions

28. Solve by Two phase simplex Method.

Minimize $2 x_{1}-3 x_{2}+6 x_{3}$
Subject to $3 x_{1}-4 x_{2}-6 x_{3} \leq 2$
$2 x_{1}+x_{2}+2 x_{3} \geq 11$
$x_{1}+3 x_{2}-2 x_{3} \leq 5$
$x_{1}, x_{2}, x_{3} \geq 0$
29. Solve by simplex method.

Maximise $x_{1}+x_{2}+x_{3}$
Subject to $2 x_{1}+x_{2}+2 x_{3} \leq 2$
$4 x_{1}+2 x_{2}+x_{3} \leq 2$
$x_{1}, x_{2}, x_{3} \geq 0$
30. Solve the transportation problem. Food bags have to be lifted by 3 different types of aircraft $A_{1}, A_{2}, A_{3}$ from an airport and dropped in flood affected villages $V_{1}, V_{2}, V_{3}, V_{4}, V_{5}$. The quantity of food that can be carried in one trip by aircraft. $A_{i}$ to village $v_{j}$ is given in the following table. The total no: of trips that $A_{i}$ can make in a day is given in the last column .The no: of trip possible each day to village $v_{i}$ is given in the last row. Find the no: of trips each aircraft should make on each village so that the total quantity of food transported in a day is maximum.

|  | $V_{1}$ | $V_{2}$ | $V_{3}$ | $V_{4}$ | $V_{5}$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $A_{1}$ | 10 | 8 | 6 | 9 | 12 | 50 |
| $A_{2}$ | 5 | 3 | 8 | 4 | 10 | 90 |
| $A_{3}$ | 7 | 9 | 6 | 10 | 4 | 60 |
|  | 100 | 80 | 70 | 40 | 20 |  |

31. If the arrivals are completely random. Show that the probability distribution of no: of arrivals in fixed time interval follows a Poisson distribution.
