Reg. No.. Name.. **19U642**

B. Sc. DEGREE END SEMESTER EXAMINATION - MARCH 2019

SEMESTER **– 6: MATHEMATICS (CORE COURSE)**

COURSE: **15U6CRMAT12: LINEAR ALGEBRA AND METRIC SPACES**

(Common for Regular - 2016 Admission / Supplementary-Improvement 2015/2014 admissions)

Time: Three Hours Max. Marks: 75

PART A

Answer ***all*** questions. Each question carries ***1*** mark

1. Define vector space.
2. Give an example of an infinite dimensional vector space.
3. Let W be a nonzero subspace of a vector space V of dimension 4. What can you say about the dimension of W?
4. Define T: R → R by T(x) = x+2. Is *T* linear?
5. Linear transformation T: R^2 –→ R^2 is such that T(0,1) = (0,0) and T(1,0) = (0,2) .Find T(x,y).
6. What do you mean by the nullity of a linear transformation?
7. Define metric. Give an example of a metric on R.
8. Give an example of an open subset of R which is not an interval.
9. What is a complete metric space?
10. What is a dense set? Give an example. (1 x 10 = 10)

PART B

Answer ***any Eight*** questions. Each question carries ***2*** marks

11. Can you find a set of three vectors of R^2 which is linearly independent? Justify.
12. What is the span of a set of vectors? If a vector space is spanned by n vectors, what can you say about its dimension?
13. Is it possible to extend the set {*(1, 0,-1), (0,-1, 2)*} to a basis of R^3?
14. Prove that the range of a linear transformation *T* : *V* → *W* is a subspace of *W*.
15. Let A be an $m \times n$ matrix over R. Can you define a linear transformation from R^n into R^m using A?
16. Define the rank and nullity of a linear transformation.
17. Differentiate between open sphere and open set.
18. Show that singleton set is a closed subset of a metric space.
19. When do you say that a function between two metric spaces is continuous? Give an example of a function which is continuous at every point of the domain.
20. What is the difference between limit and limit point of a sequence? (2 x 8 = 16)

PART C

Answer ***any Five*** questions. Each question carries ***5*** marks

21. Show that a linearly independent subset of a vector space can be extended to a basis of the space.
22. Find the dimension of the space of all 2×2 matrices by establishing a basis.
23. Let V be an n – dimensional vector space and let T be a linear transformation from V into V such that the range and null space of T are identical. Prove that n is even. Find such a T.
24. Let $T : R^2 \to R^2$ be defined by T(x,y) = (-y, x). Prove that T is linear. Find the matrix of T in the standard basis
{*(1, 0), (0, 1)*} of R^2.
25. Let X be a metric space with metric d. Show that d_1 defined by $d_1(x, y) = d(x, y)/(1 + d(x, y))$ is also a metric on X. Prove that (X, d_1) is a bounded metric space.
26. In a metric space prove that intersection of a finite number of open sets is open. What can you say about arbitrary intersection of open sets?
27. Let X and Y be metric spaces. If f: X → Y is continuous, prove that for any sequence $(x_n) \to x$ in X implies $(f(x_n)) \to f(x)$ in Y. (5 x 5 = 25)

PART D

Answer ***any Two*** questions. Each question carries ***12*** marks

28. (a) Let V be the real vector space of all functions f from R into R. Which of the following sets of functions are subspaces of V? (a) all f such that f(0) = f(1) (b) all f such that f(3) = 1+ f(-5) (c) all f such that f(-1) = 0 (d) all f which are continuous.
(b) Prove that the n rows of an invertible matrix form a basis for R^n.

29. (a) State and prove the rank - nullity theorem for linear transformations.
(b) Let $T: R^3 \to R^3$ defined by T(x, y, z) = (3x+z, -2x+y, -x+2y+4z). Prove that T is linear. Find the matrix of T in the standard basis for R^3.

30. Define $d(X, Y) = \max\{|x_1 - x_2|, |y_1 - y_2|\}$ where $X = (x_1, y_1)$ and $Y = (x_2, y_2)$. Show that d is metric on R^2. Draw the closed sphere of radius one unit and center at the origin.

31. Let X be a metric space and Y be a complete metric space and let A be a dense subspace of X. If f is uniformly continuous mapping of A into Y prove that f can be extended uniquely to a uniformly continuous mapping g from X into Y. (12 x 2 = 24)