Reg. No…………………………………………. Name…………………………………………… **19U629**

B. Sc. DEGREE END SEMESTER EXAMINATION - MARCH 2019

SEMESTER – 6: MATHEMATICS (CORE COURSE)

COURSE: 15U6CRMAT11: DISCRETE MATHEMATICS

(Common for Regular - 2016 Admission / Supplementary-Improvement 2015 / 2014 admissions)

Time: Three Hours Max. Marks: 75

Part A

Answer **all** questions. Each question carries 1 mark.

1. Define a complete bipartite graph.
2. Draw a graph without cut vertices.
3. Let $G = (V, E)$ be a graph such that $|E| = 8$ and deg $(v) = 2$ for all $v \in V$, then find the Number of vertices of G .
4. Define a Hamiltonian graph.
5. Draw an Eulerian graph.
6. For what values of n does the complete graph K_n has a perfect matching.
7. State Knapsack problem.
8. Give an example of super increasing sequence.
9. Every chain is a lattice. Justify.
10. Define a self dual of a partially ordered set. (1 x 10 = 10)

Part B

Answer **any eight** questions. Each question carries 2 marks.

11. Prove that every $u - v$ walk contains a $u - v$ path.
12. Define adjacency matrix of a graph. Write down adjacency matrix of K_4 .
13. Draw all non-isomorphic trees with five vertices.
14. State personnel assignment problem.
15. If G be a graph in which the degree of every vertex is atleast two, then prove that G contains a cycle.
16. Prove that a simple graph G is Hamiltonian if and only if its closure $c(G)$ is Hamiltonian.
17. Explain Merkle- Hellman cryptosystem.
18. Explain monoalphabetic cipher and polyalphabetic cipher.
19. Prove that the poset $X = \{2,3,4,6\}$ of non trivial factors of 12 under divisibility is self dual.
20. Show that union of 2 sublattices may not be a sublattice. (2 x 8 = 16)

Part C

Answer **any five** questions. Each question carries 5 marks.

21. If T is a tree with n vertices then it has precisely $n - 1$ edges.
22. Define cut vertex of a graph G . If G be a graph with n vertices, $n \geq 2$, then prove that G has atleast 2 vertices which are not cut vertices.

23. Let G be a simple graph. Show that either G or its complement $\bar{G}$ is connected.
24. Prove that a connected graph G is Euler if and only if the degree of every vertex is even.
25. Define a perfect matching and prove that a tree has atmost one perfect matching.
26. Explain how encrypytion and decryption are carried out in RSA cryptosystem.
27. Prove that product of 2 lattices is a lattice. (5 x 5 = 25)

Part D

Answer **any two** questions. Each question carries 12 marks.

28. Let G be a non empty graph with atleast 2 vertices. Then prove that G is bipartite if and only if it has no odd cycles.
29. State and prove Dirac's theorem for Hamiltonian graphs.
30. (i) Explain how encryption and decryption are carried out using Hill's cipher.

    (ii) The cipher text ALXWU VADCOJO has been enciphered with cipher $C_1 \equiv 4P_1 + 11P_2$ (mod 26), $C_2 \equiv 3P_1 + 8P_2$ (mod 26). Derive the plain text.
31. (i) Prove that the dual of a lattice is a lattice.

    (ii) Show that a finite lattice has least and greatest elements. (12 x 2 = 24)

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