Reg. No... Name.. **19U616**

B. Sc. DEGREE END SEMESTER EXAMINATION - MARCH 2019

SEMESTER – **6: MATHEMATICS (CORE COURSE)**

COURSE: **15U6CRMAT10 : COMPLEX ANALYSIS**

(Common for Regular - 2016 Admission / Supplementary-Improvement 2015/ 2014 admissions)

Time: Three Hours Max. Marks: 75

Part A

Answer All Questions. Each Question carries 1 Mark.

1. Define continuity of a function at a point.
2. Write the function f(z) = $z^3 + 1$ in the form f(z) = u (x,y) + i v (x,y).
3. Show that Log(1 – i) = $\frac{1}{2}$ ln 2 - $\frac{\pi}{4}$ i
4. Define a simple closed curve and give examples.
5. If C is a simple closed contour, what is the value of $\int_C \exp(2z)\, dz$?
6. State Morera's theorem.
7. Define convergence of an infinite series of complex numbers.
8. Define essential singular point of a function.
9. Give an example of a function with a removable singular point.
10. State Jordan's Lemma.

Part B

Answer any eight questions. Each question carries 2 marks

11. Write the function f(z) = z + $\frac{1}{z}$ (z ≠ 0) in the form of f(z) = u (r, $\theta) + i\, v(r, \theta)$
12. Show that the real and imaginary parts of an analytic function are harmonic functions.
13. Find all the values of z such that e^z = - 2.
14. Evaluate $\int_C \frac{z+2}{z}$ where C is the circle z = $2e^{i\theta}$ $(\pi \le \theta \le 2\pi)$
15. Evaluate $\int_C \frac{\exp(3z)}{z^2}$ dz where C is the circle $|z|$ = 1.
16. State and prove Liouville's theorem.
17. Write the statement of Laurent's theorem.
18. Find the Taylor series expansion of $\frac{1}{z}$ about z = -1 and state the region of validity of the expansion.
19. Discuss the nature of singularity of f $(z) = \frac{1-\cos z}{z^3}$ at z = 0.
20. Determine the order of the poles and the corresponding residues for $\frac{\exp z}{z^2+\pi^2}$

Part C

Answer any five questions. Each question carries 5 marks

21. Show that u (x,y) = sinhx.siny is harmonic in some domain and find the harmonic conjugate of it.
22. Find all roots of the equation sinhz = i.
23. State and prove Cauchy's integral formula.
24. Let C be any simple closed contour, described in the positive sense on the z- plane and $g(z) = \int_C \frac{s^3+2s}{(s-z)^3}$ ds. Show that g(z) = $6\pi iz$ when z is inside C and g(z) = 0 when z is outside C.
25. Represent the function $f(z) = \frac{z+1}{z-1}$,
    a. by Maclaurin series and state where it is valid.
    b. by Laurent series in the domain $1 < |z| < \infty$
26. State and prove Cauchy's residue theorem.
27. Find the residue of :
    a. $f(z) = \frac{1-e^{2z}}{z^4}$ at z = 0
    b. $g(z) = \frac{1}{(z^2+a^2)^2}$ at z = ai

Part D

Answer any two questions. Each question carries 12 marks

28.
    a. Prove that satisfaction of Cauchy- Reimann equations is a necessary condition for f(z) = u(x,y) + i v(x,y) to be analytic in a domain S.
    b. If f(z) = u(x,y) + iv(x,y) is analytic in a domain S, show that the families of level curves u(x,y) = C_1 and v(x,y) = C_2 are orthogonal.
29.
    a. State and prove Liouville's theorem.
    b. State and prove maximum modulus principle.
30. Write the two Laurent series in powers of z that represent the function $f(z) = \frac{1}{z\,(1+z^2)}$ in certain domains and specify the domains.
31. Using residue theorem evaluate:
    a. $\int_{-\pi}^{\pi} \frac{d\theta}{1+\sin^2\theta}$ b. $\int_0^{\infty} \frac{x^2}{(x^2+1)(x^2+4)}$ dx