Reg. No.. Name.. **19U604**

B. Sc. DEGREE END SEMESTER EXAMINATION - MARCH 2019

SEMESTER – 6: MATHEMATICS (Common for BSc Mathematics / BSc Computer Applications)

COURSE: **15U6CRMAT9 /15U6CRCMT7: REAL ANALYSIS**

(Common for Regular - 2016 Admission / Supplementary/Improvement 2015 / 2014 Admissions)

Time: Three Hours Max. Marks: 75

Part A

Answer **All** Questions. Each Question has 1 Mark.

1. Is the series $\sum_{n=1}^{\infty} \cos(1/n^2)$ is convergent. Justify.
2. State Cauchy's general principle of convergence for a series.
3. Say True or False. Every convergent series is absolutely convergent. Justify.
4. Give an example of a function f on $\mathbb{R}$ such that $|f|$ is continuous at a point but f is not continuous at that point.
5. Define Uniform continuity of a function.
6. Show by an example that every bounded functions need not be Riemann integrable.
7. Let f be a function defined on $[0,1]$ as follows $f(x) = \begin{cases} 1 & ; x \neq 1/2 \\ 0 & ; x = 1/2 \end{cases}$ evaluate $\int_0^1 f(x)dx$.
8. State Fundamental theorem of Calculus.
9. Define uniform convergence.
10. State Cauchy's general principle of uniform convergence. (1 x 10 = 10)

Part B

Answer **any Eight**. Each Question has 2 Marks

11. Find the sum of the series $\sum_{k=1}^{\infty} (3/4)^{k+2}$.
12. Check whether the given series converges or not $\sum_{n=1}^{\infty} \left(n^n/n!\right)$.
13. Prove that an infinite series $\sum u_n$ converges then $\lim_{n\to\infty} u_n = 0$.
14. If f, g are continuous functions at appoint a, prove that the function $Max\{f, g\}$ is continuous at a.
15. Is the function $f(x) = x/x + 1$ uniformly continuous for $x \in [0,2]$. Justify.
16. Let f be a bounded function on $[a, b]$ and let m, M be the infimum and supremum of f on $[a, b]$, then for any partition P of $[a, b]$ prove that $m(b - a) \leq L(P, f) \leq U(P, f) \leq M(b - a)$.
17. When we say a bounded function f is Riemann integrable on $[a, b]$.
18. Show that a constant function is Riemann integrable.
19. Write a short note on point wise and uniform convergence of sequence of functions
20. Write any test for checking uniform convergence of sequence of functions.

(2 x 8 = 16)

Part C

Answer **any Five**. Each Question has 5 Marks.

21. State and prove D' Alembert's Ratio test.
22. Test the convergence of the series $1 + \frac{x}{1!} + \frac{2^2x^2}{2!} + \frac{3^3x^3}{3!} + \cdots, x > 0$.
23. If a function f is a continuous in a closed interval $[a, b]$ and $f(a), f(b)$ have opposite signs, then prove that there exist $c \in [a, b]$ such that $f(c) = 0$.
24. Examine the continuity of the function f defined by $f(x) = \frac{2[x]}{3x-[x]}$ at $x = -\frac{1}{2}$ and $x = 1$.
25. Prove that a bounded function f is integrable in $[a, b]$, if the set of its points of discontinuity is finite.
26. If f and g are two functions, both bounded and integrable in $[a, b]$ then their product fg is also bounded and integrable in $[a, b]$.
27. State and prove necessary and sufficient condition for the uniform convergence of a sequence of functions. (5 x 5 = 25)

Part D

Answer **any Two**. Each Question has 12 Marks.

28. a) Prove that the positive term geometric series $1 + r + r^2 + \cdots$ converges for $r < 1$ and diverges for $r \geq 1$.
    b) State and prove Leibnitz test.
29. a) Prove that a function f is continuous on a closed interval $[a, b]$ then it is uniformly continuous on $[a, b]$.
    b) Write an example of any function on $\mathbb{R}$ which is discontinuous only at two points.
30. a) State and prove the necessary and sufficient condition for integrability.
    b) Give an example of a function f such that $|f|$ is integrable but f is not integrable. Justify your answer.
31. a) State and prove Weierstrass -M test for uniform convergence of sequence of functions.
    b) Prove that the sequence $\{f_n\}$, where $f_n(x) = {}^{x}/_{1 + nx^2}$ converges uniformly on any closed interval I.

(12 x 2 = 24)

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