Reg. No…………………………………. Name……………………………………………………… **19U528**

B.Sc. DEGREE END SEMESTER EXAMINATION – OCTOBER 2019

SEMESTER –5: MATHEMATICS (CORE COURSE)

COURSE: 15U5CRMAT07: ABSTRACT ALGEBRA

(Common for Regular 2017 Admission & Supplementary/Improvement 2016/2015/2014 Admissions)

Time: Three Hours Max. Marks: 75

PART A

Each Question carries 1 Mark Answer All Questions

1. Show that $(E, +)$, E is the set of all non-negative even integers is a subsemigroup of (N, +), N is the set of all-natural numbers.
2. Prove that the set G = {0,1,2,3,4,5} is an abelian group under addition modulo 6.
3. What condition to be satisfied if an algebraic system (A, +, .) is called a ring?
4. Obtain the generator of the group (Z_{13}, $+_{13}$).
5. Prove that a cyclic group of order eight is homorpic to a cyclic group of order four.
6. "Every group of prime order is abelian", True or False, justify.
7. How many homomorphism are there of $\mathbb{Z}$ into $\mathbb{Z}_8$?
8. Give an example for a non-commutative ring.
9. Define integral domain.
10. Find the characteristics of the rings $\mathbb{Z}_5$ and $\mathbb{Q}$. (1 x 10 = 10)

PART B

Each Question carries 2 Marks Answer any Eight

11. If G is a finite group of even order then prove that there exists at least one element $a \neq e$ where e is the identity element, such that $a = a^{-1}$.
12. For a cyclic group of order n generated by an element $'a'$, show that 'n' is the least positive integer for which $a^n = e, e$ is the identity element.
13. Let $f = (1\ 4\ 3\ 2\ 5\ )$ and $g = (1\ 2)(4\ 3\ 5) \in\ S_5$ find fog and gof.
14. Define cosets and Lagrange's theorem.
15. Prove that a factor group of a cyclic group is cyclic.
16. Define factor group and alternating group.
17. Define zero divisors and find the zero divisors of $\mathbb{Z}_4$.
18. Using Fermat's theorem, find the remainder of 3^{47} when divided by 23.
19. If R is a ring with unity and N is an ideal of R containing a unit then prove that N = R.
20. For a given subrings $U_1\ and\ U_2$ of a ring R, show that their intersection $U_1 \cap U_2$ is also a subring of R. (2 x 8 = 16)

PART C

Each Question carries 5 Marks Answer Any Five

21. Prove that every permutation of a finite set is a product of disjoint cycles.
22. Find all subgroups of $< \mathbb{Z}_{12,} +_{12} >$.
23. Let $G = \{1, a, a^2, a^3\}$ $(a^4 = 1)$ be a group and $H = \{1, a^2\}$ is a subgroup of G under multiplication. Find all cosets of H.
24. A cyclic group G with generator a is finite if and only if there exist a positive integer k such that $a^k = e$.
25. Let G be a finite abelian group of order n, and let 'r' be a positive integer relatively prime to n, then show that $\phi_r : G \to G$ given by $a\phi_r = a^r$ is an isomorphism of G onto itself.
26. Show that $Q(\sqrt{7}) = \{a + b\sqrt{7}/a, b \ \in Q\}$ is a field under usual addition and multiplication.
27. If 'p' is prime prove that $\mathbb{Z}_p$ is a field. (5 x 5 = 25)

PART D

Each Question carries 12 Marks Answer Any Two

28. Prove that every finite group of order $'n'$ is isomorphic to a permutation group of degree$'n'$.
29. State and prove any necessary and sufficient condition for any subset of a group become a subgroup under the same group operation.
30. State and prove fundamental homomorphism Theorem.
31. (a) Prove that every field is an integral domain.

    (b) Prove that a field contains no proper nontrivial ideal. (12 x 2 = 24)

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