Reg. No………………………………. Name…………………………………………………. **19U504**

# B. Sc. DEGREE END SEMESTER EXAMINATION - OCTOBER 2019

SEMESTER **–5: MATHEMATICS (CORE COURSE FOR MATHEMATICS AND COMPUTER APPLICATION)**

COURSE: **15U5CRMAT5-15U5CRCMT5, MATHEMATICAL ANALYSIS**

*(Common for Regular 2017 admission & Improvement 2016/Supplementary 2016/2015/2014 admission)*

Time: Three Hours Max. Marks: 75

## PART A

**Answer all questions. Each question carries 1 mark.**

1. Define neighborhood of a point.
2. Check whether the set $\left\{\frac{1}{n} : n \in \mathbb{N}\right\}$ is open. Justify your claim.
3. State Bolzano-Weierstrass Theorem for sets.
4. Define bounded set with an example.
5. State Order - Completeness property of Real Numbers.
6. State true or false: A sequence can have more than one limit point. Justify your claim.
7. Find limit inferior and limit superior of the sequence $\left\{sin\frac{n\pi}{3}\right\}$.
8. State Cauchy's General Principle of convergence.
9. Reduce to a real number: $(1 - i)^4$.
10. State the binomial formula for complex numbers. (1 x 10 = 10)

## PART B

**Answer any eight questions. Each question carries 2 marks.**

11. Show that every open set is a union of open intervals.
12. Prove: The union of an arbitrary family of open sets is open.
13. Show that the set $S = \left\{1, \frac{1}{2}, -\frac{1}{2}, \frac{1}{3}, -\frac{1}{3}, \dots \dots\right\}$ is neither open nor closed.
14. Check whether the set of natural numbers is bounded above and below. Justify your claims.
15. Find the infimum and supremum of the set $\left\{1 + \frac{(-1)^n}{n} : n \in \mathbb{N}\right\}$.
16. Show that the sequence $\{(-1)^n\}$ oscillates finitely.
17. Show that if $\lim_{n\to\infty} a_n = l$, then $\lim_{n\to\infty} (a_1 a_2 \dots a_n)^{1/n} = l$.
18. Show that for any real number $x$, $\lim_{n\to\infty} \frac{x^n}{n!} = 0$.
19. Locate the numbers $z_1 + z_2$ and $z_1 - z_2$ vectorially when $z_1 = 2i$ and $z_2 = 2 - i$.
20. Define interior and exterior points of a set in complex plane. (2 x 8 = 16)

## PART C

**Answer any five questions. Each question carries 5 marks.**

21. Prove: A set is closed if and only if its complement is open.
22. Prove: The cartesian product of two countable sets is countable.

23. State and prove: Archimedean property of real numbers.
24. Prove: A monotonic sequence is convergent if and only if it is bounded.
25. State and prove the nested intervals theorem.
26. Show that $\lim_{n\to\infty} \frac{1}{n}\left[1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n}\right] = 0.$
27. Sketch the following sets and determine which are domains:
    (a) $|z - 2 + i| \leq 1$
    (b) $|2z + 3| > 4$
    (c) $Im(z) > 1$ (5 x 5 = 25)

**PART D**

**Answer any two questions. Each question carries 12 marks.**

28. (a) Prove: Every bounded infinite set has the smallest and greatest limit points.
    (b) Prove or disprove: The set of real numbers in [0, 1] is countable.
29. State the two forms of Completeness Property of real numbers and prove their equivalence.
30. (a) State and prove: Cantor's Intersection theorem for real line.
    (b) State and prove: Cauchy's First Theorem on limits.
31. (a) Find the two square roots of $\sqrt{3} + i$.
    (b) Find the principal argument of $\frac{i}{-2-2i}$ and $\left(\sqrt{3} - i\right)^6$. (12 x 2 = 24)

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