Reg. No.. Name.. **19U224**

B.Sc. DEGREE END SEMESTER EXAMINATION MARCH/APRIL 2019

SEMESTER – 2: MATHEMATICS (CORE COURSE FOR MATHEMATICS AND COMPUTER APPLICATIONS)

COURSE: 15U2CRMAT2/15U2CRCMT2– ANALYTIC GEOMETRY, TRIGONOMETRY AND MATRICES

(Common for Regular 2018/Supplementary-improvement 2017/2016/2015 /2014 Admission)

Time: Three Hours Max. Marks: 75

PART A

Answer ***all*** questions.
Each question carries 1 mark.

1. Define orthoptic locus.
2. What is the equation of the chord joining the points t_1 and t_2 on the parabola $y^2 = 4ax$?
3. Define Conjugate Diameters of an ellipse.
4. What is the equation of the polar of (x_1, y_1) with respect to the conic $y^2 = 8x$?
5. The polar coordinates of a point is $(5, \pi/6)$. Write its Cartesian coordinates.
6. Define hyperbolic sine of x
7. Separate $\sin(\alpha + i\ \beta)$ into real and imaginary parts.
8. When x is real, what is the period of $\tan x$?
9. What is the rank of a non-singular matrix of order $n \times n$.
10. What is the inverse of the elementary transformation $H_{21}(-2)$.

(10 X 1 = 10)

PART B

Answer any ***eight*** questions.
Each question carries 2 marks.

11. Find the pole of the line $lx + my + n = 0$ with respect to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$.
12. Find the condition for the line $y = mx + c$ to be a tangent to the parabola $y^2 = 4ax$ and also determine the point of contact.
13. Find the equation of the asymptotes to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$.

14. Find the equation of the tangent to the conic $\frac{l}{r} = 1 + e\cos\theta$ at a point whose vertical angle is α.

15. Prove that $\cosh^2 x = 1 + \sinh^2 x$

16. Separate into real and imaginary parts the expression $\tan(\alpha + i\beta)$.

17. If $\log(\alpha + i\,\beta) = x + iy$, find x.

18. Find the normal form of the matrix $\begin{bmatrix} 1 & 2 & -1 & 4 \\ 2 & 4 & 3 & 5 \\ -1 & -2 & 6 & -7 \end{bmatrix}$

19. Find the eign values of the matrix $\begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$

20. Give the definition of elementary matrix. Find the elementary matrix $H_{23}(4)$ obtained from I_3.

(8 X 2 = 16)

PART C

Answer any *five* questions.
Each question carries 5 marks.

21. If the chord joining two points whose eccentric angles are α and β on the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ cut the major axis at a distance d from the centre, show that $\tan\frac{\alpha}{2}\tan\frac{\beta}{2} = \frac{d-a}{d+a}$.

22. Find the equation of the pair of tangents from (x_1, y_1) to the parabola $y^2 = 4ax$.

23. Find the condition that the line $\frac{l}{r} = A\cos\theta + B\sin\theta$ may be a tangent to the conic $\frac{l}{r} = 1 + e\cos\theta$.

24. If $\sin(\theta + i\,\phi) = r(\cos\alpha + i\,\sin\alpha)$, prove that $r^2 = \frac{1}{2}(\cosh 2\phi - \cos 2\theta)$.

25. Find the sum of the series to the infinity. $\cos x \sin x + \frac{1}{2!}\cos^2 x \sin 2x + \frac{1}{3!}\cos^3 x \sin 3x + \ldots$.

26. Use Cramer's rule to solve the system of linear equations.

$$\begin{aligned} x + z &= -1 \\ -2x + y &= 1 \\ -y + z &= 5 \end{aligned}$$

27. Form augmented matrix and solve the system of equations

$$\begin{aligned} x + y + z &= 9 \\ 2x + 5y + 7z &= 52 \\ 2x + y - z &= 0 \end{aligned}$$

(5 X 5 = 25)

PART D

Answer any *two* questions.

Each question carries **12** marks.

28. (a) Show that the line $lx + my + n = 0$ is a normal to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, if $\frac{a^2}{l^2} + \frac{b^2}{m^2} = \frac{(a^2 - b^2)^2}{n^2}$.

(b) Show that the locus of the poles of normal chords of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ is the curve $y^2a^6 - x^2b^6 = (a^2 + b^2)^2x^2y^2$.

29. A circle passing through the focus of a conic, whose latus rectum is $2l$, meets the conic in four points whose distances from the focus are r_1, r_2, r_3, r_4. Prove that $\frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3} + \frac{1}{r_4} = \frac{2}{l}$.

30. (a) Factorise $x^7 - 1$ into real factors

(b) Sum the series $\cosh\alpha - \frac{1}{2}\cosh 2\alpha + \frac{1}{3}\cosh 3\alpha \ldots$ to ∞.

31. State Cayley-Hamilton Theorem. Use the theorem to compute A^3 and A^4 of the matrix $A = \begin{bmatrix} 1 & 1 & 2 \\ 3 & 1 & 1 \\ 2 & 3 & 1 \end{bmatrix}$.

(2 X 12 = 24)