Reg. No……………………………………………. Name…………………………………………… **19U142S**

B. Sc. DEGREE END SEMESTER EXAMINATION - OCTOBER 2019

SEMESTER – 1: MATHEMATICS (COMPLEMENTARY COURSE FOR PHYSICS/CHEMISTRY)

COURSE: 15U1CPMAT1: DIFFERENTIAL CALCULUS AND TRIGONOMETRY

(Common for Improvement 2018/ Supplementary 2018/2017/2016 /2015/2014 admission)

Time: Three Hours Max. Marks: 75

PART A (Short Answer Questions)

Answer **all** questions. Each question carries **1** mark.

1. What is the value of $\lim_{x\to -1} 3(2x-1)^2$?
2. State Sandwich theorem.
3. Find the derivative of $y = x^2 \sin x$.
4. The absolute maximum value of $f(x) = x^2$ on [-2,1] is __________ .
5. State Mean Value Theorem.
6. Find $\frac{\partial f}{\partial y}$ if $f(x,y) = y \sin xy$.
7. State the chain rule for functions of two independent variables.
8. Define hyperbolic sine of x.
9. Prove that $\cosh^2 x - \sinh^2 x = 1$.
10. Separate into real and imaginary parts $\sin(x + i y)$. (1 x 10 = 10)

PART B (Brief Answer Questions)

Answer **any eight** questions. Each question carries **2** marks.

11. Evaluate $\lim_{x\to 1} \frac{x^2+x-2}{x^2-x}$.
12. Find the first and second derivatives of $s = -2t^{-1} + \frac{4}{t^2}$.
13. State the first derivative theorem for local extreme values.
14. Find the value or values of c which satisfies the mean value theorem for the function $f(x) = x^2 + 2x - 1$ in the interval [0,1].
15. Find the extreme value of the function $y = x^3 - 2x + 4$ and find where they occur.
16. Determine f_x and f_y if $f(x,y) = \frac{2y}{y+\cos x}$.
17. Show that $f(x,y) = e^{-2y} \cos 2x$ satisfies Laplace's equation.
18. Using chain rule find the derivative of $w = x y$ with respect to 't' along the path $x = \cos t$, $y = \sin t$.
19. Prove that $\cos 4\theta = \cos^4\theta - 6 \sin^2\theta \cos^2\theta + \sin^4\theta$.
20. If $u = \log \tan\left(\frac{\pi}{4} + \frac{\theta}{2}\right)$, Prove that $\tanh\frac{u}{2} = \tan\frac{\theta}{2}$. (2 x 8 = 16)

PART C (Short Essay Type Questions)

Answer **any five** questions. Each question carries **5** marks.

21. Calculate the derivative of the function $f(x) = 4 - x^2$ using the definition.
22. Find $\frac{d}{dt}(\tan(5 - \sin 2t))$ using chain rule.
23. Using the first derivative test for monotonic functions find the critical points of $f(x) = x^3 - 12x - 5$ and identify the intervals on which f is increasing and decreasing.
24. Verify $w_{xy} = w_{yx}$: Given $w = x \sin y + y \sin x + xy$.
25. Evaluate $\frac{\partial w}{\partial u}$ and $\frac{\partial w}{\partial v}$ at (u,v) = (1/2 , 1), given $w = xy + yz + xz$, $x = u + v$, $y = u - v$, $z = uv$.
26. Show that $\cos(\alpha + i\beta) + i \sin(\alpha + i\beta) = e^{-\beta}(\cos\alpha + i \sin\alpha)$.
27. Prove that $\sinh^{-1} x = \log(x + \sqrt{x^2 + 1})$. (5 x 5 = 25)

PART D (Essay)

Answer **any two** questions. Each question carries **12** marks.

28. (a) Evaluate $\lim_{x \to 1} \frac{x-1}{\sqrt{x+3} - 2}$

    (b) Find the slope of the circle $x^2 + y^2 = 25$ at the point (3, -4).
29. Consider the function f whose derivative is given by $f'(x) = (x-1)(x+2)(x-3)$

    (a) what are the critical points of f?

    (b) On what interval is f increasing or decreasing?

    (c) At what points, if any, does f assume local maximum and minimum values?
30. (i) The plane $x = 1$ intersects the paraboloid $z = x^2 + y^2$ in a parabola. Find the slope of the tangent to the parabola at (1,2,5).

    (ii) State the mixed derivative theorem.

    (iii) Find $\frac{dy}{dx}$ if $xy + y^2 - 3x - 3 = 0$ at (-1,1)
31. Sum to infinity the series $c \sin\alpha + \frac{1}{2} c^2 \sin 2\alpha + \frac{1}{3} c^3 \sin 3\alpha + \ldots$

(12 x 2 = 24)

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