# B. Sc. DEGREE END SEMESTER EXAMINATION - OCTOBER 2019 SEMESTER - 1: COMPLEMENTARY FOR BSC PHYSICS / BSC CHEMISTRY COURSE -19U1CPMAT1: CALCULUS - 1 <br> (For Regular - 2019 Admission) 

Time: Three Hours
Max Marks: 75

## PART - A

Answer any 10 questions. Each question carries 2 marks.

1. State Mean Value Theorem.
2. Find the absolute extrema of $f(x)=x^{2}$ on $[-2,1]$.
3. Find the function $f(x)$ whose derivative is ' $\sin x$ ' and whose graph passes through the point [0,2].
4. Find the first order partial derivatives of $f$, when $f(x, y)=x \tan ^{-1}(x y)$.
5. Show that $z=\ln \sqrt{x^{2}+y^{2}}$ satisfy the Laplace equation.
6. Find the greatest and smallest values that the function $f(x, y)=x y$ takes on the ellipse $\frac{x^{2}}{8}+\frac{y^{2}}{2}=1$
7. Find the Volume of the solid obtained by rotating the region bounded by $y=x^{3}, y=8, x=0$ about $y$-axis
8. Find the length of the asteroid $x=\cos ^{3} t, y=\sin ^{3} t, 0 \leq t \leq 2 \pi$
9. Find the area of the surface swept out by revolving the circle $x^{2}+y^{2}=1$ about the x -axis.
10. Evaluate $\int_{0}^{1} \int_{0}^{1-y} \int_{0}^{2} d x d z d y$.
11. Write the Jacobian determinant of $x=g(u, v), y=h(u, v)$.
12. Find the average value of $f(x, y)=x(\cos x y)$ over the rectangle $0 \leq x \leq \pi$ and $0 \leq y \leq 1$.
$(2 \times 10=20)$

## PART - B

Answer any five questions. Each question carries 5 marks
13. Does the function $f(x)=\sqrt{x(1-x)}$ on $[0,1]$ satisfy the hypothesis of mean value theorem?
14. Show that $y=\tan x$ is increasing in every interval in its domain.
15. Find all local maxima, local minima and saddle point of $f(x, y)=x^{3}+y^{3}-3 x-12 y+20$
16. The plane $x+y+z=1$ cuts the cylinder $x^{2}+y^{2}=1$ in an ellipse. Find the points on the ellipse that lie closest to and farthest from the origin.
17. Find the area of the region enclosed by $x+y^{2}=0$ and $x+3 y^{2}=2$.
18. The region bounded by $y=x^{2}, y=2-x, x=0$ for $x \geq 0$ revolves about $y$ axis. Find the volume of the solid generated by shell method.
19. Find the area of the region cut from the first quadrant by the cardioid $r=1+\sin \theta$
20. Evaluate the integral by changing the Cartesian integrals to polar integrals
$\int_{-1}^{1} \int_{-\sqrt{1-y^{2}}}^{\sqrt{1-y^{2}}}\left(x^{2}+y^{2}\right) d x d y$
$(5 \times 5=25)$

## PART - C

Answer any three questions. Each question carries 10 marks.
21.
(a) Find the critical point of $f(x)=x^{3}-12 x-15$. Identify the intervals on which f is increasing and decreasing.
(b) Show that the function $h(\theta)=3 \cos \frac{\theta}{2}, 0 \leq \theta \leq 2 \pi$ at $\theta=0$ and $\theta=2 \pi$ have local extreme values at the given values of $\theta$ and say which kind of local extreme the function has.
22. Find the absolute maximum and minimum values of $f(x, y)=2+2 x+2 y-x^{2}-y^{2}$ on the triangular region in the first quadrant bounded by the lines $x=0, y=0, y=9-x$.
23. Find the volume of the solid generated by revolving the triangle with vertices $(1,1),(1,2),(2,2)$ about
(i) the line $x=10 / 3$.
(ii) the line $\mathrm{y}=1$
24. Find the volume of the region enclosed by the surfaces $z=x^{2}+3 y^{2}$ and $z=8-x^{2}-y^{2}$
$(10 \times 3=30)$.

