Reg. No………………………………………….   Name……………………………………………   **19U126S**

B.Sc./BC.A. DEGREE END SEMESTER EXAMINATION - OCTOBER 2019

SEMESTER **– 1: MATHEMATICS (COMMON FOR MATHEMATICS / COMPUTER APPL. / B. C. A.)**

COURSE: **15U1CRMAT1-15U1CRCMT1-16U1CPCMT1: FOUNDATION OF MATHEMATICS**

(Common for Improvement 2018/ Supplementary 2018/2017/2016 /2015/2014 admission)

Time: Three Hours   Max. Marks: 75

PART A (Short Answer Questions)

*Answer **all** questions. Each question carries **1** mark.*

1. Let A and B be two sets. Show that $A \cap B \subseteq A$.
2. Let A, B and C is three sets. Show that $\overline{A \cup (B \cap C)} = (\overline{C} \cup \overline{B}) \cap \overline{A}$.
3. How many relations are there on a set with n numbers?
4. List the triples in the relation $\{(a, b, c): a, b, c \text{ } are \text{ } integers \text{ } with \text{ } 0 < a < b < c < 5\}$.
5. Define n-ARY relations.
6. Determine whether the conditional statement *"if 1+1=3 then 2+2=5"* is true or false.
7. Write the negation of "Heather will go to the concert or Steve will go to the concert"
8. State Unique Factorization Theorem.
9. Define Euler's Function.
10. Find the number of divisors of 7128.   (1 x 10 = 10)

PART B (Brief Answer Questions)

*Answer **any eight** questions. Each question carries **2** marks.*

11. What is the value of sum of the terms of the GP $\sum_{i=0}^{3} \sum_{j=0}^{2} (3i + 2j)$.
12. Let f_1, f_2 be functions from **R** to **R** such that $f_1(x) = x^2$, $f_2(x) = (x - x^2)$. Find f_1+f_2 and f_1f_2.
13. Let S = {1,2,3,4,5} and R the relation, R = {(1,3),(2,4),(3,5),(1,1),(2,2),(4,2),(3,1)}. Is this relation on S an equivalence relation? Justify.
14. Determine whether the poset ({1, 2, 4, 8, 16}, /) is a lattice and draw the Hasse diagram.
15. Define linearly ordered set with example.
16. Show that $p \rightarrow q$ and $\sim q \rightarrow \sim p$ are logically equivalent.
17. Are $\sim(p \wedge q)$ and $\sim p \vee \sim q$ logically equivalent? Justify.
18. State and prove "Little Fermat's Theorem".
19. Show that $n^7 - n$ is divisible by 42.
20. If $2^n - 1$ is a prime, Show that $2^{n-1}(2^n - 1)$ is a perfect number.

(2 x 8 = 16)

PART C (Short Essay Type Questions)

*Answer **any five** questions. Each question carries **5** marks.*

21. Prove that $\overline{A \cap B} = \overline{A} \cup \overline{B}$.
22. Let R be the relation on the set of ordered pairs of positive integers such that $(a,b),(c,d) \in R$ iff ad=bc. Show that R is an equivalence relation.
23. Suppose that the relations R_1 and R_2 on a set A are represented by the matrices

    $M_{R1} = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$, $M_{R2} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$. Write the matrices representing $R_1 \cup R_2$ and $R_1 \cap R_2$.
24. Use quantifiers to express the statement that "there exist a woman who has taken a flight on every airline in the world".
25. Show that the hypothesis "If you send me an email message, then I will finish writing the program", "If you do not send me an email message, then I will go to sleep early" and "if I go to sleep early, then I will wake up feeling refreshed" lead to the conclusion " If I do not finish writing the program, then I will wake up feeling refreshed" .
26. State and Prove Euler's Extension of Fermat's Theorem.
27. Show that every integer which is a perfect cube is of the form $7m$ or $7m \pm 1$.

(5 x 5 = 25)

PART D (Essay)

*Answer **any two** questions. Each question carries **12** marks.*

28. Let $f: X \to Y$ and let A and B be any 2 subsets of X. Then prove the following
    (i) $f(A \cup B) = f(A) \cup f(B)$
    (ii) $f(A \cap B) \subseteq f(A) \cap f(B)$
    Give an example to show that the inclusion can be proper.
29. Suppose that R and S are reflexive relations on a set A, prove or disprove each of these statements and give examples
    (i) $R \cup S$ is reflexive.
    (ii) $R \cap S$ is reflexive.
    (iii) $S \circ R$ is reflexive.
    (iv) $R - S$ is reflexive.
30.
    (i) Give a proof by contradiction of the theorem "If 3n+2 is odd, then n is odd"
    (ii) Give a direct proof to show that the product of two perfect squares is a perfect square.
31. State and prove Fundamental Theorem of Arithmetic. (12 x 2 = 24)

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