

B. Sc. DEGREE END SEMESTER EXAMINATION - OCTOBER 2019**SEMESTER – 1: MATHEMATICS (CORE)****COURSE: 19U1CRMAT1 – CALCULUS***(For Regular - 2019 Admission)*

Time: Three Hours

Max Marks: 75

PART – A(Answer **any 10** questions. Each question carries 2 marks.)

1. Find the intervals on which $f(x) = x^3$ is increasing and the intervals on which it is decreasing.
2. Find all critical points of $f(x) = 3x^{5/3} - 15x^{2/3}$.
3. Define absolute maximum and absolute minimum of a function defined on an interval. Also state extreme value theorem.
4. State Mean-Value theorem.
5. Evaluate $\lim_{x \rightarrow \pi/4} (1 - \tan x) \sec 2x$.
6. Find $\frac{dy}{dx}$ where $y = \operatorname{sech}(e^{2x})$.
7. Derive the formula for the volume of a sphere of radius r .
8. Use cylindrical shells to find the volume of the solid generated when the region R in the first quadrant enclosed between $y=x$ and $y=x^2$.
9. Find the area of the region bounded above by $y=x+6$, bounded below by $y=x^2$ and bounded on the sides by the lines $x=0$ and $x=2$.
10. Define level surfaces.
11. State the second partials test.
12. State constrained –extremum principle for two variables and one constraint.

(2 x 10 = 20)

Part –B(Answer **any 5** questions. Each question carries 5 marks.)

13. Find the absolute extrema, if any, of the function $f(x) = e^{(x^3 - 3x^2)}$ on the interval $(0, \infty)$.
14. Find a point on the curve $y=x^2$ that is closest to the point $(18,0)$.
15. Find $\lim_{x \rightarrow 0} (1 + \sin x)^{\frac{1}{x}}$.
16. Prove that $\tanh^{-1}(x) = \frac{1}{2} \ln \left(\frac{1+x}{1-x} \right)$, $-1 < x < 1$.
17. Find the area of the surface that is generated by revolving the portion of the curve $y=x^3$ between $x=0$ and $x=1$ about the x -axis.
18. Find the arc length of the curve $y = x^{2/3}$ from $x = 1$ to $x = 8$.

19. Given that $Z=e^{xy}$, $x=2u+v$, $y=u/v$. Find partial derivatives of Z with respect to x and y using chain rule.
20. Find the absolute maximum and minimum values of $f(x,y)=3xy-6x-3y + 7$ on the closed triangular region R with vertices $(0, 0)$, $(3, 0)$ and $(0, 5)$.

(5 x 5 = 25)

PART –C(Answer **any 3** questions. Each question carries 10 marks.)

21. Sketch a graph of $y= \frac{x^2-1}{x^3}$ and identify the locations of all asymptotes, intercept, relative extrema and inflection points.
22. State and prove Rolle's theorem.
23. Find the volume of the solid that results when the region enclosed by $y=\sqrt{x}$, $y=0$ and $x=9$ is revolved about the line $x=9$. Also find the volume of the solid that results when the given region above is revolved about the line $y=3$.
24. Find the points on the sphere $x^2+y^2+z^2=36$ that are closest to and farthest from the point $(1, 2, 2)$.

(10 x 3 = 30)
