Reg. No .... Name **19P4028**

MSc DEGREE END SEMESTER EXAMINATION - MARCH/APRIL 2019

SEMESTER 4 : MATHEMATICS

COURSE : **16P4MATT18EL : COMBINATORICS**

(For Regular - 2017 Admission and Supplementary - 2016 Admission)

Time : Three Hours Max. Marks: 75

Section A

Answer all the following (1.5 marks each)

1. Find the number of ways of arranging the 26 letters in the English alphabet in a row such that there are exactly 5 letters between x and y.
2. Explain Injection and bijection principle?
3. Prove by a combinatorial argument that the following number is always an integer for each $n \epsilon N$: $\frac{(3n)!}{2^n.3^n}$.
4. Prove the among any group of 7 people, there must be at least 4 of the same sex.
5. Find that any among any group of 3000 people, there are at least 9 who have the same birthday?.
6. Give the Bounds of Ramsey number
7. Let S = {1,2, ... ,100}. Find the number of integers in S which are divisible by 5.?
8. Explain generalized principle of inclusion and exclusion?
9. Define Partitions of Integers
10. Define conjugate partitions

(1.5 x 10 = 15)

Section B

Answer any 4 (5 marks each)

11. Find the number of ways to seat **n** married couples around a table in each of the following cases:
    (i) Men and women alternate;
    (ii) Every woman is next to her husband.
12. In how many ways can a committee of 5 be formed from a group of 11 people consisting of 4 teachers and 7 students if
    (i) there is no restriction in the selection?
    (ii) the committee must include exactly 2 teachers?
    (iii) the committee must include at least 3 teachers?
    (iv) a particular teacher and a particular student cannot be both in the committee?
13. Let A = {a_1, a_2, ... , a_5} be a set of 5 positive integers. Show that for any permutation a_{i1},a_{i2},a_{i3},a_{i4},a_{i5} of A, the product.
    (a_{i1} - a_1)(a_{i2} - a_2)........(a_{i5} - a_5) is always even.
14. Find the number of integer solutions to the equation $x_1 + x_2 + x_3 + x_4 = 20$; using properties? where, $1 \leq x_1 \leq 5, 0 \leq x_2 \leq 7, 4 \leq x_3 \leq 8$ and $2 \leq x_4 \leq 6$
15. Let A_1 ,A_2 , .. ,Aq be any q subsets of a finite set S. Then find $\left|\bar{A}_1 \cap \bar{A}_2 \cap . . . . . \cap \bar{A}q\right|$
16. Find the number of ways to select 4 members from the mu l ti-set M = {2 . b, 1 . c, 2 . d, 1 . e}.

(5 x 4 = 20)

Section C

Answer any 4 (10 marks each)

17.1. A permutation $x_1x_2 \ldots\ldots x_{2n}$ of the set { I, 2, ... , 2n }, where $n \epsilon N$, is said to have property P if $| x_i - x_{i+1} | = n$ for at least one i in {1, 2, ... , 2n - 1}. Show that, for each n, there are more permutations with property P than without?

OR

2. Explain Distribution Problems.

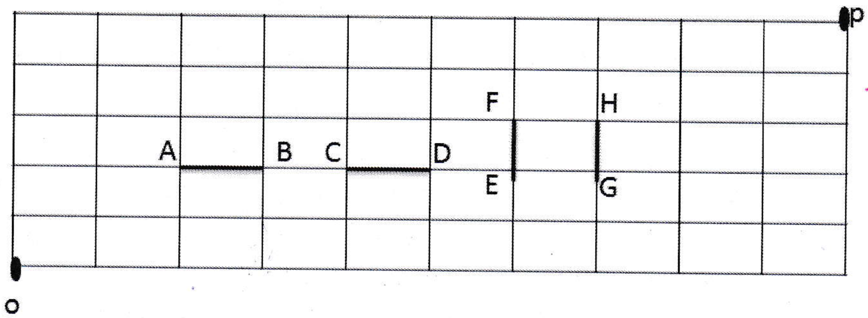
18.1. Define Ramsey number and show that R(3,3,) = 6.

OR

2. a) Prove that for all integers $p, q \leq 2$, $R(p, q) \leq R(p-1, q) + R(p, q-1)$.

b) Seventeen people correspond by mail with one another - each one with all the rest. In their letters only three different topics are discussed. Each pair of correspondents deals with only one of these topics. Prove that there are at least three people who write to one another about the same topic.

19.1.



The figure showes a 11 by 6 rectangular grid with 4 specified segments A B, CD, E F and G H. Find the number of shortest routes from O to P in each of the following cases using the method of properties:

(i) All the 4 segments are deleted;

(ii) Each shortest route must pass through exactly 2 of the 4 segments.

OR

2. Solve the recurrence relation; $a_n = a_{n-1} + a_{n-2}$, given that $a_0 = 1$ and $a_1 = 1$.

20.1. a) Solve the recurrence relation $a_n - 3a_{n-1} + 2a_{n-2} = 2n$; Given that $a_0 = 3$ and $a_1 = 8$.

b) Prove that for any n,k ϵ N, the number of partitions of n into parts, each of which appears at most k times, is equal to the number of partitions of 'n' into parts the sizes of which are not divisible by k + 1

OR

2. Let an denote the number of parallelograms contained in the nth subdivision of an equilateral triangle. Find a recurrence relation for a_n and solve the recurrence relation

(10 x 4 = 40)