Reg. No ....   Name 19P4016

MSc DEGREE END SEMESTER EXAMINATION - MARCH/APRIL 2019

SEMESTER 4 : MATHEMATICS

COURSE : **16P4MATT17EL : MULTIVARIATE CALCULUS AND INTEGRAL TRANSFORMS**

(For Regular - 2017 Admission and Supplementary 2016 Admission)

Time : Three Hours   Max. Marks: 75

Section A

Answer all the following (1.5 marks each)

1. Define Integral transforms
2. Find the Laplace transform of $\sin(at)$
3. Find the Laplace transform of $coshat$
4. Show that total derivative of a linear function is the function itself.
5. If $f(x) = ||x^2||$ then find $f'(c; u)$.
6. Let $f : \mathbb{R} \to \mathbb{R}^2$ be a function given by $f(t) = (\cos t, \sin t)$. Show that the ordinary Mean Value theorem does not hold in $[0, 2\pi]$.
7. Let $f : \mathbb{R} \to \mathbb{R}^2$ be defined by $f(t) = (\cos t, \sin t)$ and $f'(t)(u) = u(-\sin t, \cos t)$.
   Show that for every vector $a \in \exists 'z' \in (0, 2\pi)$ such that $a.\{f(y) - f(x)\} = a.\{f'(z)(y - x)\}$.
8. Show by an example that f need not be one-one on S even when $J_f(x) \neq 0 \, \forall x \in S$.
9. Write any three elementary properties of k-forms
10. Define the term primitive mapping

(1.5 x 10 = 15)

Section B

Answer any 4 (5 marks each)

11. Show that convolution may not be defined if f and g are Lebesque integrable
12. Prove that $\cos x = \frac{8}{\pi} \sum_{n=1}^{\infty} \frac{n \sin 2nx}{4n^2 - 1}$ if $0 \leq x \leq \pi$.
13. Compute the gradient vector $\nabla f(x, y)$ at those points (x, y) inR^2 where it exists.
    a) $f(x, y) = x^2 y^2 log(x^2 + y^2$ if $x, y) \neq (0, 0), f(0, 0) = 0$
    b) $f(x, y) = xysin\dfrac{1}{x^2 + y^2} if(x, y) \neq (0, 0), f(0, 0) = 0$
14. State and prove the Mean value theorem
15. Show that the rectangular solid of maximum volume that can be inscribed in a given sphere is a cube

16. Suppose $w = \sum_I b_I(x)dx_I$ is the standared representationof a $k - form\ w$ in an open set $E \in R^n$. If$w = 0$in E, then prove that $b_I(x) = 0$ for every increasing $k-$ index I and for every $x \in E$

(5 x 4 = 20)

Section C

Answer any 4 (10 marks each)

17.1. State and prove the convolution theorem for Fourier Transforms

OR

2. Find the Fourier transform of $f(x) = \begin{matrix} 1-x^2 & when|x| < 1 \\ 0 & when|x| > 1 \end{matrix}$ Use it to evaluate $\int_0^\infty \frac{xcosx - sinx}{x^3} cos\frac{x}{2}dx$

18.1. Let u and v be-two real-valued functions defined on a subset S of the complex plane. Assume also that u and v are differentiable at an interior point c of S and that the partial derivatives satisfy the Cauchy-Riemann equations at c.Then show that the function $f = u + iv$ has a derivative at c. Moreover, $f'(c) = D_1u(c) + iD_1v(c)$.

OR

2. a)Derive the matrix form of Chain rule
b)Compute the gradient vector $\nabla f(x, y)$at those points(x, y)inR^2 where it exists
$f(x, y) = xysin\frac{1}{x^2 + y^2} if f(x, y) \neq (0, 0), f(0, 0) = 0$

19.1. (a) State and prove second derivative test for extrema.
(b)Find and classify the extremum values of the function
f(x, y) = $x^2 + y^2$ + x + y + xy.

OR

2. Let $B = B(a; r)$ be an n-ball in R^n, let δBdenote its boundary,$\delta B = x : ||x - a|| = r$, and let $B = BU\delta B$ denote its closure. Let $f = (f_1, \ldots, f_n)$be continuous on B, and assume that all the partial derivatives $Djf_i(x)$ exist if $x \in B$. Assume further that $f(x) \neq f(a)$ if $x \in \partial B$ and that the Jacobian determinant $J_f(x) \neq 0$ for each x in B. Then prove that $f(B)$, the image of B under f, contains an n-ball with center at$f(a)$

20.1. Suppose E is an open set in R^n, T is a C'-mapping of E into an open set $V \subset R^m$. Let ω and λ be k- and m- forms in V respectively. Then prove that
(a) $(\omega + \lambda)_T = \omega_T + \lambda_T$ if $k = m;$
(b) $(\omega \wedge \lambda)_T = \omega_T \wedge \lambda_T;$
(c) $d(\omega_T) = (d\omega)_T$ if ω is of class C' and T is of class C''.

OR

2. Suppose ω is k-form in an open set $E \subset R^n$, ϕ is a surface in E, with parameter domain $D \subset R^k$ and Δ is the k- surface in R^k with parameter domain D, defined by $\Delta(u) = u(u \in D)$ then show that $\int_\phi \omega = \int_\Delta \omega_\phi$.

(10 x 4 = 40)