19P3043

Reg. No ..............................  Name ................................................

**M. Sc DEGREE END SEMESTER EXAMINATION - OCTOBER 2019**

**SEMESTER 3 : MATHEMATICS**

COURSE : **16P3MATT14 : OPERATION RESEARCH**

***(For Regular - 2018 Admission and Supplementary - 2016/2017 Admissions)***

Time : Three Hours  Max. Marks: 75

**Section A**

**Answer all Questions (1.5 mark each)**

1. Explain order cycle, and the two types of inventory review systems.
2. In the EOQ problem without shortage, if the set-up cost is $C_S + bQ$ instead of being fixed ($b$ = setup cost/unit item) then there is no change in the optimum order quantity produced due to change in the set-up cost: Prove the statement.
3. Define the Hessian matrix.
4. What are the necessary and sufficient conditions for a point to be local minimum?
5. Write the Kuhn-Tucker conditions for non linear optimization.
6. Describe about return function, decision variables and state transformation function?
7. Describe the examples of failure of dynamic programming?
8. What is the general form of an integer L.P.P.? Give an example.
9. Define graph and directed graph.
10. Explain the terms
 (a) Component (b) Centre (c) Strongly connected (d) Connected

(1.5 x 10 = 15)

**Section B**

**Answer any 4 (5 marks each)**

11. $f(x) = 4 - 7x + x^2$
 (a) Calculate the value of this function at $x_0 = 2$
 (b) Find the value of $f(x)$ at $x = 4$ by using (a) and the derivatives of $f(x)$.
12. Minimize $y(x) = 100(x_2 - x_1^2)^2 + (1 - x_1)^2$.
13. Minimize $u_1 + u_2 + u_3$ subject to $u_1 + u_2 + u_3 \geq 10,\ u_1, u_2, u_3 > 0$ by forward recurssion.
14. What are the characteristics of dynamic programming ?
15. Find the maximum potential difference between $v_1$ and $v_4$ in the following graph
 $v = 1\ 2\ 3\ 4$
 $u =$ (1,2) (1,3) (2,3) (3,4) (4,2) (1,4)
 subject to $-2 \leq f_2 - f_1 \leq 3, 6 \leq f_3 - f_2 \leq 10, f_4 - f_3 \leq -2,$
 $-2 \leq f_2 - f_4, 1 \leq f_4 - f_1 \leq 6, f_3 - f_1 \leq 7.$
16. Describe minimum path problem. Give an algorithm to find the minimum path when all the arc lengths are non-negative.

(5 x 4 = 20)

## Section C

**Answer any 4 (10 marks each)**

17.1. (a) Explain EOQ problem with finite replenishment for an inventory problem with shortage.
(b) The demand for an item in a company is 18000 units per year, and the company can produce the items rate of 3000 per month. The cost of one setup is Rs. 500, and the holding cost of 1 unit per month is 15 paise. The shortage cost of one unit is Rs. 20 per month. Determine

i. Optimum production batch quantity and the number of strategies.
ii. Optimum cycle time and production time
iii. Maximum inventory level in the cycle
iv. Total associated cost per year if the cost of the item is Rs. 20 per unit.

**OR**

2. (a) Explain the EOQ problems with (i) one price breaks (ii) more than one price breaks.
(b) Find the optimum order quantity for a product for which the price breaks are as follows:

| Quantity | Unit cost (Rs) |
|---|---|
| $0 \leq Q_1 < 800$ | Rs. 1.00 |
| $800 \leq Q_2$ | Rs. 0.98 |

The yearly demand for the product is 1600 units per year, cost of placing an order is Rs. 5, the cost of storage is 10% per year.

18.1. Maximize the function, $f(x) = -3x^2 + 21.6x + 1$ with a minimum resolution of 0.5 over 6 functional evaluation. The optimal value of $f(x)$ is assumed to lie in the range $0 \leq x \leq 25$.

**OR**

2. Solve the problem through classical Lagrangian technique.
(a) Minimize $f(x) = x_1^2 + x_2^2 - 4x_1 + 2x_2 + 5$ subject to $g(x) = x_1 + x_2 = 4$.
(b) Minimize $f(x) = (x_1 - 2)^2 + (x_2 - 1)^2$ subject to $g(x) = x_1 - 2x_2 + 1 = 0$.

19.1. Describe a method in dynamic programming to solve the problem. Minimize $\sum_{j=1}^{n} f_j(u_j)$

subject to $\prod_{j=1}^{n} u_j > K > 0,\ u_j \geq 0$

**OR**

2. Using D.P solve the following

$$\begin{aligned} \text{maximize} \quad & z = x_1 + 9x_2 \\ \text{subject to} \quad & 2x_1 + x_2 \leq 25 \\ & x_2 \leq 11 \\ & x_1, x_2 \geq 0. \end{aligned}$$

20.1. (a) Describe the algorithm for minimum path problem with arc length is unrestricted in sign.
(b) Find the minimum path from $v_1 \rightarrow v_8$ in the graph with arc and length as follows.

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| Arc | (1,2) | (1,3) | (1,4) | (2,3) | (2,6) | (2,5) | (3,5) | (3,4) | |
| Length | -1 | 4 | -11 | 2 | -8 | 7 | -3 | 7 | |
| Arc | (4,7) | (5,6) | (5,8) | (6,3) | (6,4) | (6,7) | (6,8) | (7,3) | (7,8) |
| Length | 3 | 1 | 12 | 4 | 2 | 6 | -10 | -2 | 2 |

**OR**

2. Maximize $|x_1 + 2|x_2$ subject to $4x_1 + 7x_2 + x_3 = 13$, $x_1, x_2, x_3$ are non negative integer using branch and bound method.

(10 x 4 = 40)