19P3031

Reg. No ............................... Name ...................................................

**M. Sc DEGREE END SEMESTER EXAMINATION - OCTOBER 2019**

**SEMESTER 3 : MATHEMATICS**

COURSE : **16P3MATT13 : GRAPH THEORY**

***(For Regular - 2018 Admission and Supplementary - 2016/2017 Admissions)***

Time : Three Hours Max. Marks: 75

## Section A
**Answer all Questions (1.5 mark each)**

1. Show that an edge $e = xy$ is a cut edge of a connected graph $G$ if and only if there exist vertices $u$ and $v$ such that $e$ belongs to every $u - v$ path in G.
2. Prove or disprove: Let G be a simple connected graph with $n \geq 3$.Then $G$ has a cut edge if and only if it has a cut vertex.
3. Define the eccentricity of a vertex $v$ of a connected graph $G$ . Also define the center of a connected graph $G$.
4. Give an example of a tree with two central vertices, one of which is also a centroidal vertex.
5. Give an example of a tree with disjoint center and centroid.
6. Define an Eulerian graph.Give examples of Eulerian and non-Eulerian graphs.
7. Define proper vertex coloring and chromatic number of a graph $G$.
8. Draw the Petersen graph
9. Explain the Jordan Curve Theorem.
10. Determine $\chi'(K_4)$

(1.5 x 10 = 15)

## Section B
**Answer any 4 (5 marks each)**

11. Give an example of a graph with κ = 1,λ = 2 and δ = 3.
12. Show that every connected graph contains a spanning tree.
13. Show that a subset $S$ of $V$,the vertex set of a graph is independent if and only if $V - S$ is a covering of G.By means of an example, show that the edge analogue of this theorem need not be true.
14. Prove that a simple k-regular graph on $2k - 1$ vertices is Hamiltonian.
15. Define a Hamiltonian-connected graph.If $G$ is a simple graph with $n \geq 3$ vertices such that $d(u) + d(v) \geq n + 1$, for every pair of non adjacent vertices of $G$, then $G$ is Hamiltonian-connected.
16. Let G be a simple plane cubic graph having eight faces. Determine n(G).
Draw two such graphs that are non-isomorphic.

(5 x 4 = 20)

## Section C

**Answer any 4 (10 marks each)**

17.1. Define automorphism of a simple graph $G$. Show that the set $\Gamma(G)$ of all automorphisms of a simple graph $G$ is a group with respect to the compositions of mappings as the group operation. Further show that for any simple graph $G, \Gamma(G) = \Gamma(G^c)$

**OR**

2. (a) Prove that for a loopless connected graph $G$, $\kappa(G) \leq \lambda(G) \leq \delta(G)$.
(b) Give an example of a graph for which $\kappa(G) < \lambda(G) < \delta(G)$.
(c) Prove or disprove: If $H$ is a subgroup of $G$, then (i) $\kappa(H) \leq \kappa(G)$.(ii) $\lambda(H) \leq \lambda(G)$.

18.1. Show that every tree has a center consisting of either a single vertex or two adjacent vertices.

**OR**

2. Find the number of spanning trees of the graph $C_3 \vee K_1$

19.1. Show that a connected graph is Eulerian if and only if it admits a cycle decomposition.

**OR**

2. Show that for every positive integer $k$, there exists a triangle-free graph with chromatic number $k$.

20.1. (a) Explain the terms plane embedding and spherical embedding.
(b) Explain stereographic projection.
(c) Show that a graph is planar if and only if it is embeddable on a sphere.

**OR**

2. Show that every planar graph is 5-vertex colorable.

(10 x 4 = 40)