Reg. No .... Name **19P2055**

MSc DEGREE END SEMESTER EXAMINATION - MARCH/APRIL 2019

SEMESTER 2 : MATHEMATICS

COURSE : 16P2MATT10 : REAL ANALYSIS

(For Regular - 2018 Admission and Supplementary - 2017/2016 Admissions)

Time : Three Hours Max. Marks: 75

Section A

Answer all the following (1.5 marks each)

1. Show that a polynomial is always a function of bounded variation on every compact interval.
2. If f is an increasing function defined on [a,b], prove that the sum of jumps of f at every finite collection of points in (a,b) is always bounded.
3. Prove that if $f \in \mathscr{R}$, then $f^2 \in \mathscr{R}$. Is converse true? Justify.
4. Suppose f is a real, continuously differentiable function on [a,b], $f(a) = f(b) = 0$ and $\int_a^b f^2(x)\,dx = 1$. Prove that $\int_a^b xf(x)f'(x)\,dx = \frac{-1}{2}$.
5. If f is Riemann integrable, then f is of bounded variation, Justify.
6. Define Uniform closure. Prove that a sequence $\{f_n\}$ converges to f with respect to the metric of $\mathscr{C}(X)$ if and only if $f_n \longrightarrow f$ uniformly on X.
7. Prove that $C(X)$ is complete with respect to the supremum norm.
8. Show by an example that the limit of a sequence of Riemann integrable functions need not be Riemann integrable.
9. Prove that $\lim\limits_{x\to 0}(1+x)^{1/x} = e$.
10. Find the limit $\lim\limits_{x\to 0} \frac{x-\sin x}{\tan x - x}$.

(1.5 x 10 = 15)

Section B

Answer any 4 (5 marks each)

11. Prove or disprove: If f is of bounded variation on $[a,b]$ and f is nowhere zero on $[a,b]$, then $\frac{1}{f}$ is of bounded variation on $[a,b]$.
12. Derive a relation between the Riemann-Stieltjes upper sums of a function corresponding to a partition and its refinement?
13. Prove that $f \in \mathscr{R}(\alpha)$ if and only if $f\alpha' \in \mathscr{R}$
14. Discuss the uniform convergence of the sequence $\{f_n\}$, where

$$f_n(x) = \frac{x^n}{1+x^n}\ ; x \in [0,1]$$

15. If $\{f_n\}$ is a sequence of continuous functions of E and if $f_n \to f$ uniformly on E, prove that f is continuous on E.

16. Prove that if $f(x) = \sum_{n=0}^{\infty} c_n x^n; \ -1 < x < 1$ and $\sum c_n$ is convergent, then $\lim_{x \to 1} f(x) = \sum_{n=0}^{\infty} c_n.$

(5 x 4 = 20)

Section C

Answer any 4 (10 marks each)

17.1. Define equivalent paths. State and prove a necessary and sufficient condition for equivalence of two paths which are one to one on its domain. Give an example for non-equivalent paths.

OR

2. a) Define the total variation of a function of bounded variation. Prove that the total variation is zero iff f is constant.
b) State and prove the additive property of total variation of a function of bounded variation.

18.1. Let $f(x) = \begin{cases} 1 & \text{if } x \in C \\ 0 & \text{if } x \notin C \end{cases}$, for all $x \in$ [0,1], where C is the Cantor set. Prove that $f \in \mathscr{R}$ on [0,1].

OR

2. a. Prove that if $f \in \mathscr{R}(\alpha)$, $\int_a^b f d\alpha = \int_a^c f d\alpha + \int_c^b f d\alpha$ for $a < c < b$.

b. Evaluate $\int_1^{10} f d\alpha$ where $f(x) = [log x]$ and α is the identity function.

19.1. Construct a real valued function which is nowhere differentiable and continuous everywhere on $\mathbb{R}$.

OR

2. Discuss the uniform convergence of the sequence $\{f_n(x)\}$, where $f_n(x) = \sqrt{n(1-x)}x^n, \ x \in (0,1)$

20.1. Introduce trigonometric functions using exponential series and hence derive any two properties of them.

OR

2. a. If $E(z) = \sum_{n=0}^{\infty} \frac{z^n}{n!}$, prove that $E(x) = e^x \ \forall x \in \mathbb{R}$

b. If z is a complex number wiht $|z| = 1$, prove that there is a unique $t \in [0, 2\pi)$ such that $E(it) = z$.

(10 x 4 = 40)