Reg. No .... Name 19P2031

MSc DEGREE END SEMESTER EXAMINATION - MARCH/APRIL 2019

SEMESTER 2 : MATHEMATICS

COURSE : **16P2MATT08 : ADVANCED COMPLEX ANALYSIS**

(For Regular - 2018 Admission and Supplementary - 2017/2016 Admissions)

Time : Three Hours Max. Marks: 75

Section A

Answer all the following (1.5 marks each)

1. Find the radius of convergence of $\sum n^P z^n$
2. Define Gamma function.
3. Prove that $\lceil z+1 = z\lceil z$
4. Define a Reimann's zeta function.
5. State Arzela's Theorem
6. What is the functional equation for Riemann's zeta function.
7. State the Mean value property of harmonic function
8. State Riemann mapping theorem.
9. Show that for any non-constant function $f(z)$, the points of period module M are isolated.
10. Define an elliptic function.

(1.5 x 10 = 15)

Section B

Answer any 4 (5 marks each)

11. Prove that the infinite product $\pi_{n=1}^{\infty}(1+a_n)$ with $1+a_n \neq 0$ converge simultaneously with the series $\sum\limits_{n=1}^{\infty} \log(1+a_n)$ whose terms represents the value of the principal branch of the logarithm.
12. Discuss the uniform convergence of the series $\sum\limits_{n=1}^{\infty} \frac{x}{n(1+nx^2)}$ for all real values of x.
13. Prove that zeta function can be extended to a meromorphic function in the whole complex plane whose only pole is a single pole at $S=1$ with residue 1.
14. Prove that an entire function of fractional order assumes every finite value infinitely many times.
15. If V is subharmonic then prove that kV is subharmonic.
16. Show that $\mathcal{P}(z)-\mathcal{P}(u)=\frac{-\sigma(z-u)\sigma(z+u)}{(\sigma(z))^2(\sigma(u))^2}$.

(5 x 4 = 20)

Section C

Answer any 4 (10 marks each)

17.1. State and prove `Laurent Theorem'

OR

2. Derive Legendre's Duplication formula

18.1. Let $\{b_r\}$ be a sequence of complex numbers with $\underset{\lim}{r \to \infty} b_r = \infty$ and let $P_r(\xi)$ be polynomials without constant term. Then Prove that there are functions which are meromorphic in the whole plane with poles at the points b_r and the corresponding singular parts $P_r\left(\frac{1}{z-b_r}\right)$. Moreover Prove that the most general meromorphic f_n of this band can be written in the form $f(z) = \sum\left[P_r\left(\frac{1}{z-b_r}\right) - P_r(z)\right] + g(z)$ where $b_r(z)$ are suitably choosen polynomials and $g(z)$ is analytic in the whole plane.

OR

2. a. Prove that $\zeta(S) = \frac{-\overline{|1-S}}{2\pi i} \int\limits_C \frac{(-z)^{S-1}}{e^z-1} dz$, where $(-z)^{S-1}$ is defined on the complement of the positive real axis as $e^{(S-1)\log(-z)}, -\pi < Im \log(-z) < \pi$

   b. Prove that zeta function can be extended to a meromorphic function in the whole complex plane whose only pole is a simple pole at $S = 1$ with residue 1.

19.1. a. Prove that a continuous function $u(z)$ which satisfies the condition $u(z_0) = \frac{1}{2\pi}\int_0^{2\pi} u(z_0 + re^{i\theta})d\theta$ is necessarily harmonic .

   b. State Harnack's Principle by proving the corresponding Harnack's inequality.

OR

2. Show that any even elliptic function with periods w_1, w_2 can be expressed in the form $C\prod\limits_{k=1}^{n} \frac{\mathcal{P}(z)-\mathcal{P}(a_k)}{\mathcal{P}(z)-\mathcal{P}(b_k)}$, provided 0 is neither a zero nor a pole. What is the corresponding form if the function either vanishes or becomes infinite at origin.

20.1. a. Show that an elliptic function without poles is a constant.

   b. Prove that the sum of the residues of an elliptic function is zero

OR

2. a. Prove that $\frac{\sigma'(z)}{\sigma(z)} = \zeta(z)$

   b. Prove that $\sigma(-z) = -\sigma(z)$

(10 x 4 = 40)