Reg. No .... Name 19P2018

MSc DEGREE END SEMESTER EXAMINATION - MARCH/APRIL 2019

SEMESTER 2 : MATHEMATICS

COURSE : **16P2MATT07 : ADVANCED TOPOLOGY**

(For Regular - 2018 Admission and Supplementary - 2017/2016 Admissions)

Time : Three Hours Max. Marks: 75

Section A

Answer all the following (1.5 marks each)

1. State Tietze characterization of normality.
2. Let S be a sub-base for a topological space X. Prove that if X is completely regular, then for each $V \in S$ and for each $x \in V$, there exist a continuous function $f : X \to [0,1]$ such that $f(x) = 0$ and $f(y) = 1$ for all $y \notin V$.
3. If the product space is locally connected, prove that each co-ordinate space is locally connected.
4. Define the evaluation function of the indexed family $\{f_i : X \to Y_i; i \in I\}$ of functions.
5. Let $(X, \mathbb{T})$ be a space. Let $\mathbb{D}$ be the indiscrete topology on X. Prove that the function $id_X : X \to X$ is $\mathbb{T} - \mathbb{D}$ continuous and that the family $\{id_X\}$ distinguishes points on X.
6. Define base and sub-base of a filter on a set X.
7. Define a filter associated with a net S in X.
8. If a space X is Hausdorff, prove that no filter on X can converge to more than one point in it.
9. In a second countable space prove that countable compactness implies sequential compactness.
10. Let $X^+ = XU\{\infty\}$ be the one point compactification of the space X. If X is compact, prove that $\{\infty\}$ is open in X^+.

(1.5 x 10 = 15)

Section B

Answer any 4 (5 marks each)

11. State Urysohn's Lemma and using it, prove that every T_4-space is Tychonoff.
12. Prove that a topological product is regular iff each co-ordinate space is regular.
13. Prove that if $\mathbb{C}$ is a locally finite family and $D = \underset{C \in \mathbb{C}}{UC}$, then $\bar{D} = \underset{C \in \mathbb{C}}{U\bar{C}}$
14. Let A be a subset of a space X and let $x \in X$. Prove that $x \in \bar{A}$ iff there exists a net in A which converges to x .

15. Define a sub-base of a filter on X. Let S be a family of subsets of X. Prove that there exists a filter on X having S as a sub base iff S has the finite intersection property.

16. If a space X is regular and locally compact at a point $x \in X$, then prove that x has a local base consisting of compact neighbourhoods.

(5 x 4 = 20)

Section C

Answer any 4 (10 marks each)

17.1. If the product space is non-empty, prove that each co-ordinate space is embeddable in it and hence prove that if a topological product is T_0, T_1, T_2 or regular, then each co-ordinate space has the corresponding property.

OR

2. Prove that a product space is locally connected iff each co-ordinate space is locally connected and all except finitelymany of them are connected.

18.1. State and prove Urysohn Embedding Theorem.

OR

2. Prove that every trivial space is a Pseudo-metric space and every Pseudo-metric space is completely regular. Also prove that a space is completely regular iff it can be embedded into a product of Pseudo-metric spaces.

19.1. Let $S : D \to X$ be a net and $\mathcal{F}$ the filter associated with it. Let $x \in X$. Prove that

a. S converge to x as a net iff $\mathcal{F}$ converges to x as a filter.
b. x is a cluster point of the net S iff x is a cluster point of the filter $\mathcal{F}$

OR

2. For a filter $\mathcal{F}$ on a set X, prove that the following statements are equivalent.

a. $\mathcal{F}$ is an ultra filter.
b. For any $A \subset X$ either $A \in \mathcal{F}$ or $X - A \in \mathcal{F}$.
c. For any $A, B \subset X, A \cup B \in \mathcal{F}$ iff either $A \in \mathcal{F}$ or $B \in \mathcal{F}$

20.1. Prove that a subspace of a locally compact, Hausdorff space is locally compact iff it is open in its closure.

OR

2. a. Prove that sequential compactness implies countable compactness
b. Prove that a first countable, countably compact space is sequentially compact.
c. Prove that in a second countable space, compactness, countable compactness and sequential compactness are all equivalent to each other.

(10 x 4 = 40)