19P1057

Reg. No .... Name ....

M Sc DEGREE END SEMESTER EXAMINATION - OCTOBER 2019

SEMESTER 1 : MATHEMATICS

COURSE : **16P1MATT05 : COMPLEX ANALYSIS**

(For Regular - 2019 Admission and Supplementary - 2016/2017/2018 Admissions)

Time : Three Hours Max. Marks: 75

Section A

Answer any 10 (1.5 marks each)

1. " The set M of all Mobius transformations does not have the commutative property"True or false. Justify
2. Let $f : C \to C$ be a complex valued function given by $f(z) = u(x, y) + iv(x, y)$.Suppose that$v(x, y) = 3xy^2$. Then $f$cannot be holomorphic on C for any choice of$u$. True or Faulse. Justify?
3. Prove that the map $w = \bar{z}$ is not conformal
4. Define winding number
5. Evaluate $\int_r xdz$, where r is the directed line seqment from 0 to $1 + i$
6. Find the value of $\int_C \frac{1}{z} dz$ where C is the circle $z = e^{i\emptyset}, 0 \leqq \emptyset \leqq \pi$
7. Find the types of singularities and their order of the function $\frac{1}{zsinhz}$
8. Explain the term chain
9. State Schwarz reflection principle
10. Find the residue at $z = 0$ of the function $(f(z) = \frac{e^{2z}}{z^4}$

(1.5 x 10 = 15)

Section B

Answer any 4 (5 marks each)

11. If$T(z) = \frac{az + b}{cz + d}$, find z_2, z_3, z_4 (in terms ofa, b, c, d) such that $T(z) = (z, z_2, z_3, z_4)$
12. Give a precise definition of a single valued branch of $\sqrt{z}$ and prove that it is analytic
13. State and prove Cauchy's Integral formula
14. Find zeros and discuss the singularities of the function $f(z) = \frac{z-2}{z^2} sin(\frac{1}{z-1})$
15. State Rouche's theorem and apply it to determine the number of roots of the equation $z^8 - 4z^5 + z^2 - 1 = 0$
16. If $u(x, y) = x^3 - 3xy^2 + 3x^2 - 3y^2 + x$, then prove that u is a harmonic function and find its harmonic conjugate.

(5 x 4 = 20)

Section C
Answer any 4 (10 marks each)

17.1. Prove that the cross ratio is real iff the four points lie on a circle or on a straight line. Also find the linear transformation which carries $z = \infty, i, 0$ into $w = 0, i, \infty$

OR

2. Find the Mobius transformation which maps the circle $|z| \leqq 1$ on $|w-1| \leqq 1$ and makes the points $z = 0, 1$ correspond to $w = 1/2, 0$ respectively

18.1. a. State and prove the lemma for higher derivatives
b. State and prove Liouvillie's theorem

OR

2. a. Evaluate $\int_{|z|=5} \frac{z+5}{z^2-3z-4} dz$
b. State and prove Morera's theorem

19.1. State and give the topological and analytic proof of Maximum principle

OR

2. Define a simply connected region . Also prove that a region Ω is simply connected iff $n(\Upsilon, a) = 0$ for all cycles Υ in Ω and all point a which do not belong to Ω

20.1. Evaluate $\int_0^{\pi} \frac{d\theta}{a^2 + sin^2\theta}$ where $a > 0$

OR

2. Evaluate $\int\_0^{\infty} \frac{x^2}{(x^2+a^2)^3} dx$

(10 x 4 = 40)