19P1044

Reg. No .... Name ....

MSc DEGREE END SEMESTER EXAMINATION - OCTOBER 2019

SEMESTER 1 : MATHEMATICS

COURSE : **16P1MATT04 : ORDINARY DIFFERENTIAL EQUATIONS**

(For Regular - 2019 Admission and Supplementary - 2016/2017/2018 Admissions)

Time : Three Hours Max. Marks: 75

Section A

Answer any 10 (1.5 marks each)

1. Show that the vector functions $\varphi(t) = \begin{bmatrix} e^t \\ 2e^t \end{bmatrix}, \psi(t) = \begin{bmatrix} e^{3t} \\ 4e^{3t} \end{bmatrix}$ are linearly independent on any interval $a \leq t \leq b$.
2. Transform the linear system $t\frac{dx}{dt} = ax + by$ into a linear system with constat coefficients.
$$t\frac{dy}{dt} = cx + dy$$
3. Does there exist any homogeneous linear system of two unknown functions on an interval $-\pi/2 \leq t \leq \pi/2$ such that its wronskian of two solutions is W(t) = sin(t) on $-\pi/2 \leq t \leq \pi/2$. Justify your answer.
4. Find interval of convergence of the series $\sum_{j=1}^{\infty} \frac{x^j}{j}$.
5. Determine the nature of the singularity of the point $x = 0$ of the differential equation $x^3y'' + (sinx)y = 0$.
6. The sequence of functions $\{sin(nx)\}_{n=1}^{\infty}$ is orthonormalized with respect to the weight function $r(x) = 1$ on the interval $0 \leq x \leq \pi$. State true or false and justify your answer.
7. The sequence of functions $\{cos(nx)\}_{n=1}^{\infty}$ is orthonormalized with respect to the weight function $r(x) = 1$ on the interval $0 \leq x \leq \pi$. State true or false and justify your answer.
8. Find $L[x^6 sin^2(3x) + x^6 cos^2(3x)]$ and $L[10]$.
9. Find $L[4sin(2x) + 6x]$.
10. Calculate $\int_0^{\infty} \frac{sinx}{x} dx$.

(1.5 x 10 = 15)

Section B

Answer any 4 (5 marks each)

11. Consider the vector functions $\varphi(t) = \begin{bmatrix} t \\ 1 \end{bmatrix}$ and $\psi(t) = \begin{bmatrix} te^t \\ e^t \end{bmatrix}$.Show that the constant vectors $\varphi(t_0)$ and $\psi(t_0)$ are linearly dependent for each t_0 in the interval $0 \leq t \leq 1$, but the vector functions φ and ψ are linearly independent on $0 \leq t \leq 1$.
12. Find the general solution of the system $\frac{dx}{dt} = 5x - 2y$, $\frac{dy}{dt} = 4x - y$.
13. Find and classify the singular points of the differential equation $x^3(x-1)y'' - 2(x-1)y' + 3xy = 0$.

14. Explain Strum - Liouville problem and properties of its characteristic values and characteristic functions.
15. Find the Laplace transform of $xsin3x$.
16. Use Laplace transform to solve the differential equation $y'' - 4y' + 4y = 0$ with initial conditions $y(0) = 0$ and $y'(0) = 3$.

(5 x 4 = 20)

Section C
Answer any 4 (10 marks each)

17.1. Find particular solution of the linear system $\frac{dx}{dt} = 3x + 5y$, $\frac{dy}{dt} = -2x + 5y$ that satisfies the initial conditions x(0) = 5 and y(0) = -1.

OR

2. Find the general solution of the homogeneous linear system $\frac{dx}{dt} = \begin{bmatrix} 3 & -1 \\ 4 & -1 \end{bmatrix} x.$

18.1. Find general solution of $y'' + y = 0$ directly and by using method of power series.

OR

2. Use the method of Frobenius series to solve the differential equation $2x^2y'' + x(2x+1)y' - y = 0$ about the regular singular point 0.

19.1. Consider the Strum-Liouville problem $\frac{d}{dx}\left[p(x)\frac{dy}{dx}\right] + [q(x) + \lambda r(x)]\,y = 0$ with boundary conditions $A_1y(a) + A_2y'(a) = 0$ and $B_1y(b) + B_2y'(b) = 0$ where A_1, A_2, B_1, B_2 are real constants such that A_1 and A_2 are not both zero and B_1 and B_2 are not both zero. Show that the characteristic funcions corresponding to distinct characteristic values are orthogonal with respect to the weight function $r(x)$ on the interval $a \leq x \leq b$.

OR

2. Find characteristic functions of the Strum - Liouville problem $\frac{d^2y}{dx^2} + \lambda y = 0, y(0) = 0, y(\pi) = 0$ and show that the characteristic funcions corresponding to distinct characteristic values are orthogonal with respect to the weight function $r(x) = 1$ on the interval $0 \leq x \leq \pi$.

20.1. Use Laplace transform to find at least one solution of the differential equation $xy'' + (3x-1)y' - (4x+9)y = 0$.

OR

2. Describe Abel's Mechanical Problem and derive Abel's integral equation.

(10 x 4 = 40)