19P1017

Reg. No .... Name ....

M. Sc DEGREE END SEMESTER EXAMINATION - OCTOBER 2019

SEMESTER 1 : MATHEMATICS

COURSE : **16P1MATT02 : BASIC TOPOLOGY**

(For Regular - 2019 Admission and Supplementary - 2016/2017/2018 Admissions)

Time : Three Hours Max. Marks: 75

Section A

Answer all Questions (1.5 mark each)

1. Give an example to show that a topological space need not be a metric space.
2. Consider the ususl topology on R, write any three neighbourhood of the point 2.
3. Consider $A = [0, 1)$ as a subset of R with usual topology. Find int(A) and cl(A)?
4. Define i^{th} projection and its subbase.
5. Define a compact set A in a space $(X, \mathscr{T})$. Give an example of a set that is not compact.
6. Consider R with usual topology. Check whether R is Lindeloff.
7. Is the set of rational numbers connected ? Justify.
8. Prove that every non empty connected subset is contained in a unique component.
9. Define (i) Tychnoff space (ii) Regular Space.
10. Define normal space. Give an example.

(1.5 x 10 = 15)

Section B

Answer any 4 (5 marks each)

11. Define neighbourhood and show that a subset of a topological space is open if and only if it is a neighbourhood of each of its points.
12. Prove that every second countable space is separable.
13. Define topological invariant. Establish four equivalent condition for a function to be homeomorphism.
14. Show that every path connected space is connected. Is the converse true ? Justify your answer.
15. Show that closure of a connected space is connected. Is the converse true. Justify your answer.
16. Prove that normality is a weakly hereditary property.

(5 x 4 = 20)

Section C

Answer any 4 (10 marks each)

17.1. State and prove the closure axiom on a topological space $(X, \mathscr{T})$.

OR

2. (a)Let X be a set, $\mathscr{T}$ a toplogy on X and S a family of subsets of X. Show that S is a subbase for $\mathscr{T}$ if and only if S generates $\mathscr{T}$.
(b) If $(X, \mathscr{T})$ is second countable and $Y \subset X$, then show that any cover of Y by members of $\mathscr{T}$ has a countable subcover.

18.1. Show that a subset C is path component of a space X if and only if C is maximal subset of X with respect to the property of being path connected.

OR

2. Prove that product topology on $R \times R$ coincides with the metric topology on $R \times R$.

19.1. In a space X, the binary relation defined by letting $x\tilde{y}$ for $x, y \in X$ iff there exist a path in X from x to y, is an equivalence relation.

OR

2. (a) Prove that every closed and bounded interval is compact. (b) Show that union of collection of connected subsets of X having a common point is connected.

20.1. (a)Show that the following statements are equivalent in a topological space X:
i. X is regular.
ii.For any $x \in X$ and an open set G containing x, there is an open set H containing x such that $\overline{H} \subset G$.
iii.The family of all closed neighborhoods of any point of X forms a local base at that point.
(b) Show that every regular Lindeloff space is normal.

OR

2. (a)Define Tychnoff space and show that every Tychnoff space is T_3.
(b) If A, B be compact subsets of topological spaces X, Y respectively and W be an open subset of $X \times Y$ containing the rectangle $A \times B$, then prove that there exists open sets U, V in X, Y respectively such that $A \subset U, B \subset V$ and $U \times V \subset W$.

(10 x 4 = 40)