Reg. No... Name.. **18U636**

B. Sc. DEGREE END SEMESTER EXAMINATION - MARCH 2018

SEMESTER – 6: MATHEMATICS (CORE COURSE)

COURSE: 15U6CRMAT12: LINEAR ALGEBRA AND METRIC SPACES

Common for Regular (2015 Admission) & Supplementary (2014 Admission)

Time: Three Hours Max. Marks: 75

PART A

Answer ***all*** questions. Each question carries ***1*** mark.

1. Is $\boldsymbol{R}$ over $\boldsymbol{C}$ is a vector space. Justify.
2. Give an example of an infinite dimensional vector space.
3. Find the dimension of the vector space $\{(x, y, z, w) \in R^4 \; such\; that\; w = x + z = y = z + w\}$.
4. Give an example of a nonlinear transformation from $R^2 \to R$.
5. Find the null space of the identity transformation from any vector space to itself.
6. Show that if T is a linear transformation on any vector space V then $T(0) = 0$ where $0 \in V$.
7. Describe the interior of Cantor set.
8. Show by an example prove that arbitrary intersection of open sets in a metric space need not open.
9. Define convergence of a sequence in a metric space.
10. Give an example of a no where dense set. (1 x 10 = 10)

PART B

Answer ***any eight*** questions. Each question carries ***2*** marks.

11. Is union of two sub spaces of a vector space is a sub space. Justify.
12. Determine whether if $\{u, v\}$ is linearly independent in a vector space V then so too is $\{u + v, u - v\}$.
13. Prove that a subset of a vector space consisting of a single vector 'v' is linearly dependent if and only if $v = 0$.
14. Let $T: V \to W$ be a linear transformation, show that $\boldsymbol{Ker(T)}$ is a subspace of V.
15. Prove or dis prove the map $T: R^2 \to R$ defined by $(x, y) = xy$; $(x, y) \in R^2$ is linear or not.
16. Let $T: V \to W$ be a linear transformation. Prove that T is 1-1 if and only if $T(x) = 0$ implies $x = 0$.
17. Verify that the map $d(x, y) = |x^2 - y^2|$, for any $x, y \in R$ is a metric on R.
18. Is the set of natural numbers $\boldsymbol{N}$ is open in the metric space $\boldsymbol{R}$ of real numbers. Justify.
19. Let X and Y are metric spaces and f is a mapping of X into Y. If f is a constant mapping show that f is continuous. Use this show that a continuous mapping need not have the property that the image of every open set is open.
20. Let *X* and *Y* be metric spaces and f a uniformly continuous mapping from *X* to *Y*. Show that the image of a Cauchy sequence in *X* is a Cauchy sequence in *Y*. (2 x 8 = 16)

PART C

Answer ***any five*** questions. Each question carries ***5*** marks.

21. i) Let V be the set of pairs (x, y) of real numbers and F be the field of real numbers. Define $(x, y) + (x_1, y_1) = (x + x_1, 0)$ and $c(x, y) = (cx, 0)$; $c \in F$. Is V with these operations is a vector space.

ii) Determine the co-ordinate representation of the matrix $\begin{bmatrix} 4 & 3 \\ 6 & 2 \end{bmatrix}$ with respect to the basis $B = \{\begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}\}$.

22. i) Determine whether $C = \{t^2 + 2t - 3, t^2 + 5t, 2t^2 - 4\}$ is a basis for the set of all polynomials of degree at most three ($\boldsymbol{P^3}$).

    ii) Let $S = \{v_1, v_2, \dots v_n\}$ is a basis for V then prove that any set consisting of more than $'n'$ vectors is linearly dependent.

23. i) Let $T: \boldsymbol{R^6} \to \boldsymbol{W}$ be a linear transformation where $\boldsymbol{R^6}, \boldsymbol{W}$ are vector spaces over **R**, also $S = \{Te_2, Te_4, Te_6\}$ spans W. Prove that Kernel (T) contains more than one element.

    ii) Let $S: V \to V$ and $T: V \to V$ be two linear transformations and define their product as another transformation from *V* into *V* defined by $(ST)v = S(Tv)$ for all v in V. This product first applies to T a vector and then S to that result. Prove that the transformation ST is linear.

24. Let V be the vector space of all polynomials of degree at most three. Let $T: V \to V$ be the linear transformation given by $T\big(p(x)\big) = p^{|}(x)$ where $p^{|}(x)$ is the derivative of $p(x)$. Find the matrix of the linear transformation T relative to the basis$\{1, x, x^2, x^3\}$.

25. Prove that in any metric space X each open sphere is an open set. What about the converse. Justify.

26. i) Give an example of an infinite class of closed sets whose union is not closed.

    ii) Give an example of a set which is both open and closed.

    iii) Neither open nor closed.

    iv) Contains a point which is not limit point of the set.

    v) Contains no point which is not a limit point of the set.

27. Let *X* and Y be metric spaces and f is a mapping from X into Y. Show that f is continuous at x_0 if and only if $x_n \to x_0$ then $f(x_n) \to f(x_0)$. (5 x 5 = 25)

PART D

Answer ***any two*** questions. Each question carries ***12*** marks.

28. i) Show that the set of all solutions of the matrix equation $Ax = 0$ where A is an $m \times n$ matrix is a subspace of $\boldsymbol{R^n}$.

    ii) Prove that every basis of a finite dimensional vector space contains same number of elements.

29. Define the nullity and rank of a linear transformation T. For a linear transformation $T: V \to W$, prove that $Rank(T) + Nullity(T) = dimV$. Hence prove that, if $dimV = dimW$, then *T* is one to one if and only if *T* is onto.

30. i) Prove that every non empty open set on the real line is the union of a countable disjoint class of open intervals.

    ii) Show that a subset of a metric space is bounded if and only if it is non empty and is contained in some closed sphere.

31. i) State and prove Baire's Theorem.

    ii) State and prove Cantor's Intersection Theorem. (12 x 2 = 24)

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