Reg. No…………………………………………. Name…………………………………………… **15U625**

B.Sc. DEGREE END SEMESTER EXAMINATION MARCH 2018

SEMESTER **– 6: MATHEMATICS (CORE COURSE)**

COURSE: **15U6CRMAT11: DISCRETE MATHEMATICS**

For Regular (2015 Admission) & Supplementary (2014 Admission)

Time: Three Hours Max. Marks: 75

PART A

Answer ***all*** questions. Each question carries **1** mark.

1. How many different spanning trees are there for a complete graph K_4 .
2. Is it possible to construct a graph with 12 vertices such that 2 of the vertices have degree 3 and the remaining vertices have degree 4. Justify.
3. Draw Peterson graph.
4. State Hall's marriage theorem.
5. Give an example of a maximum matching graph which is not perfect.
6. Draw a Hamilton graph which is not Euler.
7. Define a monoalphabetic cipher.
8. Encrypt the message HOME using the Caesar cipher.
9. State absorption laws of lattice.
10. Define a complete lattice. (1 x 10 = 10)

PART B

Answer ***any eight*** questions. Each question carries ***2*** marks.

11. Define isomorphism of graphs with an example.
12. In any graph G , show that the number of vertices with odd degree is even.
13. Draw the graph having $\begin{bmatrix} 1 & 2 & 1 \\ 2 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}$ as adjacency matrix.
14. Define a maximal non Hamiltonian graph with an example.
15. State Chinese postman problem.
16. Prove that a tree has atmost one perfect matching.
17. Using the linear cipher $C \equiv 5p + 11 \ (mod\ 26)$, encrypt the message THE MOON IS MADE OF CREAM.
18. Explain public key cryptography.
19. Draw a diagram representing the poset $\{3,5,9,15,24,45\}$ under the divisibility relation.
20. Prove that in a lattice the distributive in equality $a \wedge (b \vee c) \geq (a \wedge b) \vee (a \wedge c)$ holds for any a,b,c. (2 x 8 =16)

PART C

Answer ***any five*** questions. Each question carries ***5*** marks.

21. Prove that a graph G is connected if and only if it has a spanning tree.
22. Prove that an edge e of a graph G is a bridge if and only if e is not part of any cycle in G.
23. Show that the number of vertices of a self complementary graph is either $4m$ or $4m + 1$ for some integer m.
24. Prove that a connected graph G has am Euler trail if and only if it has atmost 2 odd vertices.
25. Prove that a matching M in a graph G is a maximum matching if and only if G contains no M-augmenting path.
26. Explain how encryption and decryption are carried out using Hill's cipher with an example.
27. Show that a poset $(L, \leq)$ is a lattice if and only if every non empty finite subset of L has Sup and Inf. (5 x 5 = 25)

PART D

Answer ***any two*** questions. Each question carries ***12*** marks.

28. State and prove Whitney's theorem.
29. For a connected graph G , prove that the following statements are equivalent:

    (i) G is Eulerian

    (ii) The degree of each vertex of G is even (iii) G is an edge disjoint union of cycles.
30. (i) Explain Knapsack cryptosystem.

    (ii) The cipher text message produced by the knapsack cryptosystem employing the super increasing sequence 1,3,5,11, 35 modulus m =73 and multiplier a =5 is 55,15,124,109,25,34 .
31. Obtain the plain text message.

    (i) Prove that the dual of complete lattice is complete.

    (ii) Prove that in any lattice L, $(a \wedge b) \vee (b \wedge c) \vee (c \wedge a) \leq (a \vee b) \wedge (b \vee c) \wedge (c \vee a)$ for all $a, b, c \in L$. (12 x 2 = 24)

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