BSc DEGREE END SEMESTER EXAMINATION MARCH 2017

SEMESTER **– 6 : BSc MATHEMATICS (CORE COURSE)**

COURSE: **U6CRMAT10 – COMPLEX ANALYSIS**

Time: Three Hours Max. Marks: 75

Part A

Answer All Questions. Each Question carries 1 Mark.

1. Define analytic functions
2. Express the function $f(z) = z^2 + z + 1$ in the form $f(z) = u(x,y) + iv(x,y)$
3. Show that $\exp(z + \pi i) = - \exp(z)$
4. Define an arc on the complex plane and give examples.
5. State Cauchy-Goursat theorem.
6. What is the value of $\int_C tan\ z\ dz$, where C is the circle $|z| = 1$
7. What is meant by convergence of an infinite series of complex numbers?
8. Define singular point of a function.
9. Define pole type singularity.
10. Give an example of a function with an essential singular point.

Part B

Answer any Eight questions. Each question carries 2 marks

11. For $f(z) = \bar{z}$ show that $f'(z)$ does not exist at any point..
12. If a function f(z) is continuous and nonzero at a point z_0, prove that $f(z)\neq 0$ throughout some neighborhood of that point.
13. Find all values of z such that exp(2z-1)=1
14. Evaluate $\int_C \frac{z+2}{z}$ dz where C is the semicircle $z = 2e^{i\theta}$ $(0\leq \theta \leq \pi)$
15. Evaluate $\int_C \frac{\exp(2z)}{z^4}$ dz where C is the circle $|z| = 1$
16. State and prove fundamental theorem of algebra.
17. State Laurent's theorem.
18. Find the Taylor series expansion of $\frac{1}{z}$ about z = 1 and state the condition of validity of the expansion.
19. Discuss the nature of singularity of $\frac{1-\exp(2z)}{z^4}$ at z=0.
20. Determine the order of the pole and the correspondence residue for $\left(\frac{z}{2z+1}\right)^3$

Part C

Answer any Five questions. Each question carries 5 marks

21. Show that u(x,y) = $\frac{1}{2}$log $(x^2 + y^2)$ is a harmonic function and find a harmonic conjugate v(x,y) of u.
22. Show that $|sinz|^2$ = $\sin^2$x + $\sinh^2$y
23. State and prove Cauchy's integral formula.
24. State and prove Cauchy's inequality.
25. Obtain the two Laurent series in powers of z that represent the function f(z) = $\frac{1}{z(1+z^2)}$ in certain domains and specify those domains.
26. State and prove Cauchy's residue theorem.
27. Using residue theorem evaluate $\int_0^\infty \frac{cos(ax)}{x^2+1}$ dx , (a>0)

Part D

Answer any Two questions. Each question carries 12 marks

28. Prove that the function f(z) =u(x,y) +iv(x,y), z=x+iy is analytic in a domain S iff u and v are continuous functions having continuous partial derivatives satisfying the differential equation $u_x = v_y$ and $u_y = -v_x$.
29. Let C be the unit circle z = $e^{i\theta}$ ($-\pi \leq \theta \leq \pi$).Then show that for real constant a, $\int_C \frac{e^{az}}{z}$ dz = $2\pi i$. Using this derive the formula $\int_0^\pi e^{a\cos\theta}\cos(a\sin\theta)\, d\theta = \pi$.
30. (a) State and prove Taylor's theorem.

    (b) Obtain the Maclaurin series expansion of f(z) = $\frac{z+1}{z-1}$ and state the region of validity.
31. Using residue theorem evaluate

    a. $\int_0^{2\pi} \frac{d\theta}{5+4\sin\theta}$

    b. $\int_0^\infty \frac{dx}{x^4+1}$