B Sc DEGREE END-SEMESTER EXAMINATION MARCH 2017

SEMESTER - 6: B. Sc. MATHEMATICS (CORE COURCE)

COURCE: U6CRMAT9, U6CRCMT7 - REAL ANALYSIS

Time: 3 hours Max.marks: 75

Part - A

(Each question has one mark. Answer all questions)

1 Show that $\sum \frac{n}{n+1}$ is not convergent

2 Show that the series 1+$\frac{1}{2}+\frac{1}{2^2}+\frac{1}{2^3}$+…………converges

3 State Leibnitz test for an alternating series

4 Give an example of a function defined on R which is discontinuous at every point

5 Define discontinuity of second kind

6 If P_1 and P_2 are two partitions of $[a, b]$ find a partition P_0 of $[a, b]$ such that it is a refinement of P_1 and P_2

7 State a necessary and sufficient condition for the intgrability of a bounded function

8 Evaluate $\int_0^2 [x]$ dx where $[.]$ denotes the greatest integer function

9 State Weierstrass's M- test for uniform convergence

10 Distinguish between Point wise convergence and Uniform convergence

Part - B

(Each question has 2 marks. Answer any eight)

11 Show that the series $\sum \frac{1}{n}$ does not convergence

12 Test for convergence $\frac{1}{3}+\frac{\sqrt{2}}{5}+\frac{\sqrt{3}}{7}+\frac{\sqrt{4}}{9}+$ ……………………..

13 If $\sum u_n$ is a positive term series such that $\lim_{n\to\infty} n(\frac{u_n}{u_{n+1}}$ - 1) = l then prove that $\sum u_n$ diverges if l $>$ 1

14 Show that f(x)= $\frac{x-|x|}{2}$; x≠ 0 and f(x)=2 at x=0 has a discontinuity of first kind at x=0

15 Prove that f(x)= x^2 is uniformly continuous in $[-1,1]$

16 Show that $f(x) = \begin{cases} 0; if\ x\ is\ rational, \\ 1; if\ x\ is\ irrational, \end{cases}$ is not integrable in $[a, b]$

17 Show that every continuous function is integrable

18 If f is continuous & integrable on $[a, b]$ show that ∋ a number c between a & b such that

$\int_a^b f(x)dx$ = (b-a)f(c)

19 Show that $\{f_n\}$ where f_n (x) = $\tan^{-1}$ (nx) ; x> 0 is uniformly convergent on $[a, b]$; a> 0

20 Test the series $\sum_1^\infty \frac{Sin\ nx}{n^p}$ for uniform convergence; p > 0

Part - C

(Each question has 5marks. Answer any 5)

21 Examine the convergence of the series $\sum_1^\infty \sqrt{\frac{n}{n+1}}\, x^n$

22 Prove that every absolutely convergent series is convergent. Also prove that the exponential series $1+\frac{x}{1!}+\frac{x^2}{2!}+\frac{x^3}{3!}+$……………….converges absolutely for all x

23 If a function is continuous on$[a, b]$, then show that it attains its bounds at least once in $[a, b]$

24 State and prove Fixed value theorem

25 If f_1 and f_2 are two bounded and integrable functions on$[a, b]$, then prove that f_1+f_2 is also integrable on $[a, b]$

26 State and prove fundamental theorem of integral calculus

27 Show that the sequence $\{f_n\}$where $f_n(x) = \frac{nx}{1+n^2x^2}$ is not uniformly convergent on any interval containing zero

Part - D

(Each question has 12marks. Answer any 2)

28 (a) State and prove the limit form of comparison test

(b) Show that $\frac{1}{1^p}-\frac{1}{2^p}+\frac{1}{3^p}-\frac{1}{4^p}+\cdots$ … … … … … … …converges for p$>$ 0

29 (a) If a function is continuous in a closed interval, prove that it is bounded therein

(b) Examine the continuity of f(x) = $\frac{e^{\frac{1}{x}}-1}{e^{\frac{1}{x}}+1}$; x$\neq$ 0 and f(x)=0 ,x=0 at the point x=0

30 (a) If a function is monotonic on $[a, b]$ then prove that it is integrable on $[a, b]$

(b) If f and g are integrable on $[a, b]$ then fg is integrable on $[a, b]$. Prove.

31 (a) State and prove Abel's test for uniform convergence

(b) Show that $\sum \frac{x}{n^p+x^2n^q}$ converges uniformly over any finite interval for p>1and, q$>$ 0